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A hybrid method for traveltime computation

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ABSTRACT

A 2D hybrid method for the computation of multi–valued traveltime maps is presented. It combines finite difference (FD–) eikonal solvers with wavefront construction (WFC), thus, taking advantage of both the efficiency of FD–methods and the ability of ray methods to compute later, higher energetic arrivals. Basic idea of the hybrid method is the fast FD–computation of all first arrivals, the automatic detection and bounding of regions where later arrivals occur, and the final application of WFC, or, alternatively, again a FD–eikonal solver, restricted to the bounded regions. The applicability of the hybrid method to complex, weakly smoothed models is demonstrated. Depending on the number and the extensions of wavefront foldings, its computational speed varies between the computational speed of the applied FD–eikonal solver and of WFC. Therefore, the hybrid method provides an important tool for pre-stack Kirchhoff migration.

INTRODUCTION

FD–solutions of the eikonal equation are widely used for a fast computation of traveltimes, e.g. for a pre–stack Kirchhoff migration. However, it is well known that these methods are restricted to first arrival traveltimes. This is a severe drawback for the migration of complex models where later arrival traveltimes should be taken into account because they usually carry higher energy. Kinematic ray tracing (KRT) has to be performed in this situation. KRT has found an efficient implementation by the method of wavefront construction (WFC) which was first developed by (Vinje et al., 1993). A further reference for modification, extension and application of WFC is, e.g., (Ettrich and Gajewski, 1996).

We propose a new hybrid method (Ettrich and Gajewski, 1997) using Vidale's FD–eikonal solver (Vidale, 1988) and WFC, thus, combining the computational speed of Vidale–method with the ability of WFC to compute later arrivals.

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IMPLEMENTATION OF THE HYBRID METHOD

The method is described using a test model of a discontinuous velocity gradient $v(z) = a + b(z - z_i)$ with $a = 2.0\,km/s,\ b = 1/s$ and $z_i = 0\,km$ for $0\,km \leq z < 1km$ and $a = 3.0\,km/s,\ b = 3/s$ and $z_i = 1\,km$ for $1\,km \leq z \leq 2\,km.$ Complete multivalued wavefronts for this model are displayed in Fig. 1a. The hybrid method consists of three steps:

Step 1: Vidale's FD–eikonal solver is used to compute first arrival traveltimes represented by first arrival wavefronts in Fig. 1b. Points with discontinuous traveltime gradient indicate regions where slower parts of the wavefronts are cut–off (arrows in Fig. 1b). These points can be detected since the slowness vector changes rapidly here.

Step 2: All points of discontinuous traveltime gradient in Fig. 1b are due to the same triplication. The discontinuity point which is closest to the source, i. e. where the wavefront starts triplicating, is chosen to define the rays belonging to the neglected slower wavefront branches. Therefore, the method of steepest descent is used to compute rays backwards through the traveltime grid to the source. These rays start at points A_1 and A_2 (see Fig. 1c) which enclose the first discontinuity point. The take–off angles at the source for both rays are then determined. They have to be known with high accuracy. Therefore, one or two steps of 2–point ray tracing from the source to points A_1 and A_2 follow to adjust the previously determined rays and their take–off angles accurately.

Step 3: Take–off angles for the rays reaching points A_i are γ_i, $i = 1, 2$. Both rays computed in Step 2 belong to different branches of the wavefronts which are separated by the neglected slower parts of the triplications. Now it is obvious that the slower wavefront branches consist of rays emitted within an angle interval $\Delta\gamma = \gamma_2 - \gamma_1$ at the source. Finally, WFC is performed for rays within this angle interval (Fig. 1c), resulting in the missing slower parts of the wavefronts.

Advantage of this algorithm is that WFC is used only to compute the later events which can not be treated with the eikonal solver. WFC is not applied to first arrival events, a task which is by far more efficiently done with Vidale's method, resulting in a hybrid method with considerable speed–up of computational time compared to pure WFC.

APPLICATION

Fig. 3 shows an application to a weakly smoothed version of the Marmousi model (Fig. 2). Vidale's method is used first to compute first arrival traveltimes. A point of discontinuity is detected and the region of the neglected propagating reverse branch is bounded.

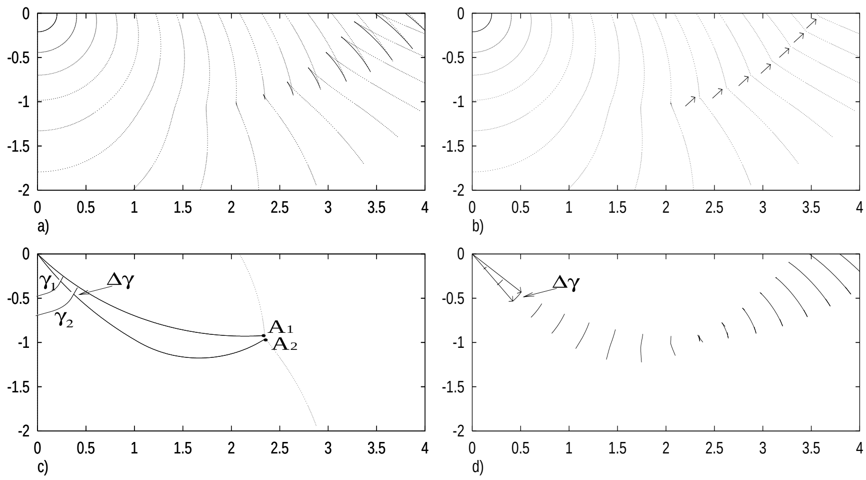

Figure 1: Complete multivalued wavefronts (a) for the test model with discontinuous velocity gradient. b)–d)Three steps of the hybrid method: First arrival wavefronts (b), rays bounding the region (c) within WFC is performed to compute later arrivals(d). Compare wavefront parts in d) with solid wavefront branches in a).

Here, instead of applying WFC in the third step of the hybrid method, we use the method by (Podvin and Lecomte, 1991), i.e., a FD-eikonal solver which is more flexible than Vidale's method.

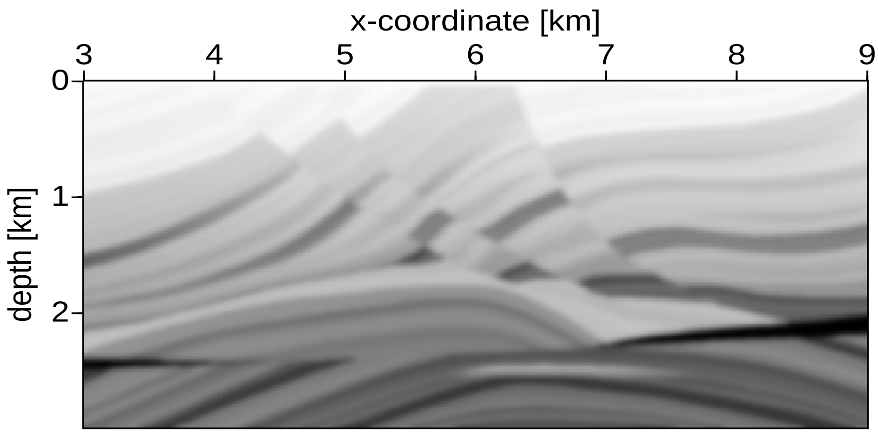

Figure 2: Weakly smoothed Marmousi–model.

CONCLUSION

The advantage of the hybrid method is the computation of all first arrival traveltimes with a FD–eikonal solver which is, at least in smooth media, faster than WFC. In this work, we use Vidale's method which is one of the fastest traveltime tools available, but

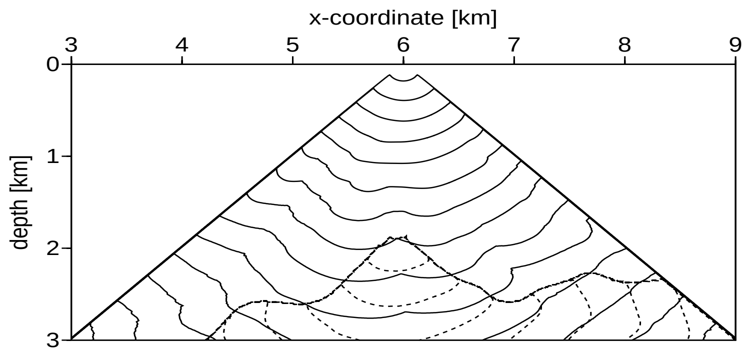

Figure 3: First arrival wavefronts (solid lines) in model of Fig. 2 computed by Vidale's FD–eikonal solver in Step 1 of the hybrid method; wavefronts of the bounded reverse branch computed by the method by (Podvin and Lecomte, 1991) in Step 3 (dashed lines).

also other FD–eikonal solvers could be used. WFC, or alternatively, again a FD–eikonal solver is applied in Step 3 of the hybrid method in regions where it is necessary owing to the occurance of later arrivals. The additional expense for the determination of boundary rays is small. The computational advantage of the hybrid method depends extremely on both the model and the source position. If there are lots of triplications making computations of Step 3 necessary the hybrid method is hardly faster than pure WFC. If, in contrast, wavefronts remain single–valued, only Vidale's method has to be applied which is much faster than WFC. Therefore, the main advantage of the hybrid method is (without user's intervention) the automatic determination of regions of wavefront folding where the important higher energetic later arrivals have to be computed in Step 3.

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