Wave Inversion Technology, Report No. 1, pages 93-97

3D finite difference post-stack time and depth remigration

J. Mann and M. S. Jaya[1]

keywords: *post-stack processing, imaging*

ABSTRACT

Post-stack remigration based on seismic image wave theory has been implemented with a refined finite difference (FD) scheme. Remigration can be performed in the depth as well as in the time domain. Post-stack constant velocity migrated sections in the appropriate domain serve as input data. Zero-offset (ZO) sections or common-midpoint (CMP) stack sections may be considered as time-migrated sections for migration velocity zero. The remigration was applied to various 2D synthetic and real data sets and a 3D ZO section of the Marmousi 3D overthrust model.

SEISMIC IMAGE WAVES

Similar to the common scalar wave equation, the so-called image wave equations [(Hubral et al., 1996)] describe a kind of propagation of imaged seismic reflectors in a fictitious image space. Generalized to 3D, these equations are given by partial differential equations (PDE)

$$4\frac{\partial^2}{\partial v \partial t} p(x,y,t,v) + vt\,\tilde{\nabla}^2 p(x,y,t,v) = 0 \tag{1}$$

$$\tilde{\nabla}^2 p(x,y,z,v) + \frac{\partial^2}{\partial z^2} p(x,y,z,v) + \frac{v}{z}\frac{\partial^2}{\partial v \partial z} p(x,y,z,v) = 0 \,. \tag{2}$$

Eq. (1) refers to the time domain, eq. (2) to the depth domain, $\tilde{\nabla}^2 = \frac{\partial^2}{\partial x^2}$ in 2D, $\tilde{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in 3D, resp. In the 4D (x,y,t,v) or (x,y,z,v) image space, each slice $v = const$ represents a migrated 3D image cube.

IMPLEMENTATION

The corresponding initial value problems $p(x,y,t,v_0) \rightarrow p(x,y,t,v)$ in the time and $p(x,y,z,v_0) \rightarrow p(x,y,z,v)$ in the depth domain are solved with semi-explicit FD schemes. Eq. (3) exemplarily shows one of the used schemes in the time domain for $v_0 < v$. The

[1]**email:** jmann@gpirs1.physik.uni-karlsruhe.de

actually used FD scheme depends on the direction of propagation and the demanded accuracy of the derivation operators. Boundary values were handled due to the zero slope condition.

$$
\begin{aligned}
p^{l+1}_{i,j,k} &= \frac{t\,\Delta t\,v\,\Delta v}{48}\left[\frac{16\left(p^l_{i,j-1,k}+p^l_{i,j+1,k}\right)-p^l_{i,j-2,k}-p^l_{i,j+2,k}-30\,p^l_{i,j,k}}{\Delta x^2}\right.\\
&+ \left.\frac{16\left(p^l_{i,j,k-1}+p^l_{i,j,k+1}\right)-p^l_{i,j,k-2}-p^l_{i,j,k+2}-30\,p^l_{i,j,k}}{\Delta y^2}\right]\\
&+ p^{l+1}_{i+1,j,k}-p^l_{i+1,j,k}+p^l_{i,j,k} \qquad (3)
\end{aligned}
$$

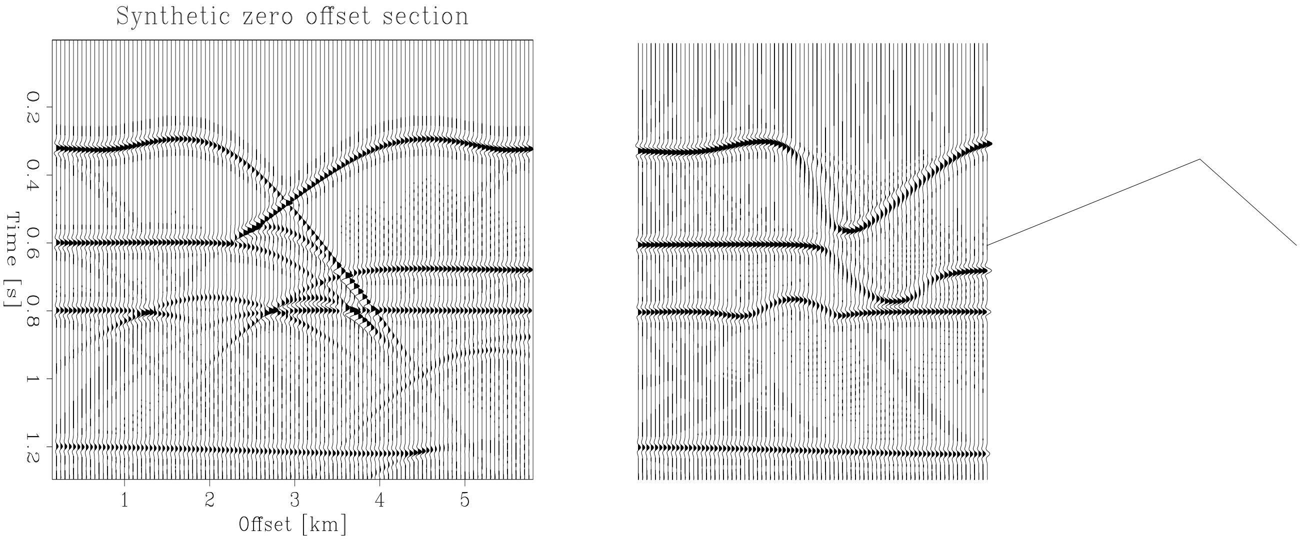
The index i denotes time, j and k offsets in x- and y-direction, resp., l velocity. Δx, Δy, Δt and Δv are the intervals in the respective direction. For forward propagation ($\Delta v > 0$), time has to be decreased in the actual calculation to achieve a stable process, and vice versa for backward propagation ($\Delta v < 0$).

APPLICATION

In order to test the intrinsic consistency of this method synthetic post-stack sections for constant velocity models were used. Figure 1 a) shows one of these sections. Figure 1 b) shows a snapshot ($v = 5$ km/s) of the remigration result generated by forward propagating the migrated image from $v = 0$ km/s to $v = 6$ km/s. Finally, figure 2 shows a pseudo ZO section generated by backward propagating the previously remigrated image from $v = 6$ km/s to $v = 0$ km/s. The result for the Marmousi 3D overthrust model is shown in Figure 3 a) and b). Since the underlying velocity model is quite inhomogeneous, the method fails to image deep reflectors with complex overburden. FD remigration was also applied to a 2D time-migrated section of a real data set provided by DMT in Bochum, Germany.

CONCLUSIONS

The FD time and depth remigration based on the image wave theory can transform migrated images in the respective domain to new ones for a continuum of constant velocities. It results as a part of one and the same algorithm eq. (1) or eq. (2), resp. Post-stack sections can be time-remigrated without any preceding migration process. Demigration of time-migrated sections is feasible by backward remigration. The method implies a constant velocity model and therefore necessarily fails when applied to inhomogeneous models. As shown in the 2D real data sets, the remigration may still lead to reasonable results in weakly inhomogeneous models. In strongly inhomogeneous models, remigration may be used in an iterative process: Using remigration to estimate the velocity in the uppermost layer, updating the velocity model, performing migration with the updated velocity model and repeating these steps for all layers.


Synthetic zero offset section
Time [s]
0.2
0.4
0.6
0.8
1
1.2
1
2
3
4
5
Offset [km]

(a)