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A unified Born–Kirchhoff representation for acoustic media

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ABSTRACT

For the modeling of a single target reflector in a smooth inhomogeneous elastic anisotropic media, it has been recently shown that the volume Born integral can be transformed into a surface scattering integral on the reflector. This surface integral has been called Born-Kirchhoff as it relates very naturally to the Kirchhoff-Helmholtz integral, thus providing the theoretical link between the two approaches. Here we specialize the derivation and main properties of the Born-Kirchhoff integral in the acoustic case, and use a simple synthetic example to provide a comparison between the new integral and its classical counterparts.

INTRODUCTION

The Born (volume) and Kirchhoff-Helmholtz (surface) representation integrals are the most widely used descriptions of reflected and transmitted wavefields due to smooth interfaces (see, e.g., Bleistein, 1984; Frazer and Sen, 1985; Lumley and Beydoun, 1993; Wapenaar and Berkhout, 1993; Chapman and Coates, 1994; Druzhinin, 1994, and Tygel et al., 1994).

Although representing basically the same phenomena, the two integrals result from quite independent formulations, and are traditionally kept as completely separate objects. Moreover, besides their fundamental distinction as volume and surface integrals (Wapenaar and Berkhout, 1993), the representations of Born and Kirchhoff-Helmholtz present also other differences, namely (a) Born assumes weak medium perturbations, uses a linearized scattering coefficient and the resulting integral is reciprocal and (b) Kirchhoff-Helmholtz imposes no contrast restrictions for the medium inhomogeneities, approximates the reflected field and its normal derivative on the reflector using the plane-wave reflection coefficient and the incident field, and the resulting integral is non reciprocal.

Under the application of a generalized form of the divergence theorem as presented in Bleistein (1984) we follow the lines of Spencer et al. (1995), de Hoop and Bleistein (1996) and Ursin and Tygel (1997), to transform the Born volume integral into a

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corresponding surface scattering integral. This new integral, called the Born-Kirchhoff integral by Ursin and Tygel (1997) in the context of elastic, anisotropic media, provides the natural theoretical link between the Born and Kirchhoff-Helmholtz representations.

In this work, we provide a quick derivation of the Born-Kirchhoff integral for the case of acoustic inhomogeneous media, as well as summarize its main properties. Furthermore, we examine its application to a simple synthetic example to compare the obtained results with the ones corresponding to its classical counterparts. We also take into the comparison a modified, reciprocal, Kirchhoff-Helmholtz integral introduced by Deregowski and Brown (1983).

We consider two unbounded, inhomogeneous acoustic media, separated by a smooth interface Σ. We also consider a reference medium characterized by smooth compression modulus $k(\mathrm{x})$ and smooth mass density $\rho(\mathrm{x})$, where $\mathrm{x} = (x, y, z)$ denotes the location vector in a fixed, global Cartesian coordinate system. The model parameters of the upper m medium coincide with those of the reference medium. The lower medium has perturbed parameters $k(\mathrm{x}) + \Delta k(\mathrm{x})$ and $\rho(\mathrm{x}) + \Delta\rho(\mathrm{x})$. The total acoustic pressure due to a point source located at $\mathrm{x}^s = (x^s, y^s, z^s)$ in the upper medium, is denoted in the frequency domain by $P(\mathrm{x}, \omega; \mathrm{x}^s)$. It satisfies the acoustic wave equation.

For observation points in the upper medium, the total pressure field can be decomposed into the superposition $P(\mathrm{x}, \omega; \mathrm{x}^s) = P^I(\mathrm{x}, \omega; \mathrm{x}^s) + P^R(\mathrm{x}, \omega; \mathrm{x}^s)$, where P^I is the *incident wavefield* and P^R is the *scattered or reflected wavefield*.

It is our aim to discuss various approximate integral representations for the reflected pressure $P^R(\mathrm{x}, \omega; \mathrm{x}^s)$ at a given receiver position $\mathrm{x}^r = (x^r, y^r, z^r)$. We first briefly review the two classical ones of Born (volume integral) and Kirchhoff-Helmholtz (surface integral). We present two alternative representations both given as surface integrals.

CONCLUSION

We have investigated the relationship between the Born volume representation integral and its counterpart Kirchhoff-Helmholtz surface integral for the model of two acoustic inhomogeneous media separated by a curved reflector. Following similar results recently provided by Ursin and Tygel (1997) for elastic, anisotropic media, we have transformed the Born volume integral into a surface Born-Kirchhoff integral of the same form as the classical Kirchhoff-Helmholtz integral. This integral provides the desired link between the two approaches. Still another representation, the Reciprocal-Kirchhoff surface integral has been presented, following a heuristic suggestion of Deregowski and Brown (1983).

For a simple model of an anticlinal reflector between two homogeneous acoustic media, we have computed the different seismograms corresponding to the described four integral representations. For the same model, we also computed, as a reference, the corresponding seismogram using a central time and central space second-order, finite-differences scheme. Some discussion and comments on the obtained results were also provided.

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PUBLICATIONS

Detailed results were presented at the Karlsruhe Workshop on Amplitude-Preserving Seismic Reflection Imaging and published in the Special Issue of Journal of Seismic Exploration (Novais et al., 1997).