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2.5-D True-Amplitude Kirchhoff Migration to Zero Offset in Laterally Inhomogeneous Media

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ABSTRACT

The proposed new Kirchhoff-type true-amplitude migration to zero-offset (MZO) for 2.5-D common-offset reflections in 2-D laterally inhomogeneous layered isotropic earth models does not depend on the reflector curvature. It provides a transformation of a common-offset seismic section to a simulated zero-offset section in which both the kinematic and main dynamic effects are correctly accounted for. The process transforms primary common-offset reflections from arbitrary curved interfaces into their corresponding zero-offset reflections automatically replacing the geometrical-spreading factor. In analogy to a weighted Kirchhoff migration scheme, the stacking curve and weight function can be computed by dynamic ray tracing in the macro-velocity model which is supposed to be available. In addition, it is shown that an MZO stretches the seismic source pulse by the cosine of the reflection angle of the original offset reflections. The proposed approach quantitatively extends the previous MZO or dip moveout (DMO) schemes to the 2.5-D situation.

INTRODUCTION

The importance of preserving seismic reflection amplitudes in seismic processing, imaging, and inversion is widely recognized. As a result, amplitude-preserving imaging methods have been developed that encompass a whole spectrum of seismic imaging procedures. One of the aims of the efforts to preserve amplitudes is the extraction and inversion of angle-dependent reflection coefficients at selected points on a target reflector. The best domain to extract this information appears to be the seismic image after pre-stack migration (Beydoun et al., 1993), e.g., from common-offset sections.

Pre-stack common-offset migration in laterally inhomogeneous media cannot be replaced by standard post-stack migration, as the stacking process, although improving the signal-to-noise ratio, destroys the quantitative amplitude information contained in the data. Moreover, it is cheaper to perform a zero-offset migration than a common-offset migration in an amplitude-preserving way. For that reason, a process is desirable that

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transforms a seismic common-offset section into its corresponding simulated zero-offset section whereby amplitudes (i.e., geometrical-spreading factors) are correctly accounted for. The true-amplitude Kirchhoff MZO described in this paper is a seismic reflection-imaging process that is exactly designed to achieve this aim.

Of course, a true-amplitude Kirchhoff-type MZO must be designed in such a way that a subsequent true-amplitude Kirchhoff zero-offset migration leads to the same result as a direct pre-stack common-offset migration applied to the original data. We recall that a true-amplitude Kirchhoff migration is defined to remove the geometrical-spreading loss from seismic reflection amplitudes (Newman, 1985; Bleistein, 1987; Bortfeld and Kiehn, 1992; Schleicher et al., 1993; Sun and Gajewski, 1997). As a consequence, true-amplitude migration outputs can be used as a measure of the local (angle-dependent) reflection coefficients. The two-step process consisting of true-amplitude Kirchhoff MZO and zero-offset migration works as follows. The MZO operation automatically replaces the geometrical-spreading factors of the common-offset reflections by the ones pertaining to the corresponding zero-offset reflections, keeping all other factors affecting the common-offset reflection amplitudes (in particular the reflection or transmission coefficients) unchanged. The zero-offset migration thereafter eliminates the zero-offset geometrical-spreading in the migration output.

For constant-velocity media, MZO can be decomposed into a two-step process, consisting of first applying a normal-moveout correction (NMO) followed by a dip-moveout correction (DMO). As the latter operation does not depend on the constant velocity and is easily implemented, this decomposition is, for the purpose of finding the correct velocity, routinely incorporated into the seismic processing sequence. It remains a good approximation as long as the velocity variation is not large. Forel and Gardner (1988) have shown that it is advantageous to invert this order, i.e., to first carry out the velocity-independent DMO before performing a conventional velocity analysis using a slightly modified NMO. In inhomogeneous media with stronger lateral variations of the velocity, however, neither of the above decompositions is properly defined. MZO has to be carried out as a one-step procedure.

MZO and DMO for constant-velocity media are widely investigated and used, leading to valuable results even for slightly varying velocities. As pointed out by Hale (1984), one great advantage of these methods is that a single 2-D operation suffices to kinematically describe a full 3-D constant-velocity DMO or MZO, thus saving a lot of computer time. This advantage, however, turns out to be a drawback when the dynamic problem is addressed. Although a full 3-D true-amplitude MZO for laterally inhomogeneous media can be derived along the lines of Tygel et al. (1996), this turns out not to be a stable process. Because of the above indicated collapse of dimensions, the ray-theory weights become zero when the out-of-plane medium variations vanish.

As an alternative to the full 3-D description, we have considered here the corresponding 2.5-D problem. In this way, we obtain a substantial extension of the constant-velocity case while avoiding the indicated difficulties. As it is now common use in exploration seismics, the term “2.5-D” means that we consider 3-D wave propagation in a 2-D (isotropic, laterally inhomogeneous, layered) earth model. There exist no medium variations in the out-of-plane y-direction. In particular, all reflectors can be specified by in-plane (x, z)-curves (Figure 1). Finally, all point sources, assumed to omnidirectionally emit identical pulses, and all receivers, assumed to have identical characteristics, are distributed along the x-axis so that only in-plane propagation needs to be considered. For the 2.5-D problem, the full 3-D geometrical-spreading factor of an in-plane ray can be written as a product of in-plane and out-of-plane factors (Bleistein, 1986). Both quantities can be computed using 2-D dynamic ray tracing (Cervený, 1987).

For a constant-velocity medium, we analytically subdivide the derived true-amplitude Kirchhoff MZO into NMO plus DMO. The resulting true-amplitude DMO transformation is then compared to some previously described time-domain smear-stack DMOs. As the principal interest of this paper is in the dynamics of MZO and DMO, we confine this comparison to the ones that explicitly address the effects on amplitudes. For that reason, several important papers on DMO, whose main emphasis are on kinematic aspects, are left out. We have focused our attention on the Born-DMO of Bleistein (1990) and Liner (1991) and the true-amplitude DMOs of Black et al. (1993) and of Fomel and Bleistein (1996). The amplitude effects of constant-velocity MZO and DMO on reflections from curved reflectors have been recently investigated by Bleistein and Cohen (1995), Goldin and Fomel (1995), and Fomel and Bleistein (1996). Their results are quite similar to ours, but obtained under the explicit use of the constant-velocity assumption.

A comprehensive analysis on true-amplitude, constant-velocity DMO has already been given by Black et al. (1993) who derived true-amplitude DMO weights for the case of primary reflections due to planar, dipping reflectors. Moreover, synthetic and field data examples of the application of the above two-step common-offset migration strategy have been provided in the same paper. The problem of a depth-dependent velocity has been addressed more recently. Dietrich and Cohen (1993) derived the analytic expression for the stacking curve in a medium with a constant vertical velocity gradient. They also heuristically suggested a DMO weight function which, however, does not correctly take into account the amplitude effect of the stacking process itself. Artley and Hale (1994) extended constant-velocity DMO to the case of a velocity that may vary arbitrarily with depth. Already in this case, ray tracing needs to be performed through a given velocity model to numerically compute the stacking curves. Their work only addresses the kinematic aspects of v(z) DMO and does not contain any considerations on amplitudes.

This paper generalizes what has been done so far in the literature in two ways. It (1) presents a 2.5-D true-amplitude Kirchhoff MZO that is designed to work for any 2-D laterally inhomogeneous layered medium for which 3-D wave propagation can be adequately described by zero-order ray theory and it (2) investigates its correctness, concerning not only the kinematic but also the dynamic, i.e., true-amplitude, aspects of primary reflections from arbitrarily curved reflectors. In analogy to a weighted Kirchhoff migration scheme, the stacking curve and weight function can be computed by dynamic ray tracing in the available macro velocity model.

At first sight, it may seem that, once the macro-velocity model is sufficiently accurate, it makes more sense to directly implement in one step a pre-stack true-amplitude common-offset migration and not perform a true-amplitude MZO at all. However, even if the main idea is to obtain a full pre-stack depth-migrated image, it may still be advantageous to perform the two-step procedure consisting of a true-amplitude MZO followed by a zero-offset migration because (a) the spatial extent of an MZO stacking curve is very limited in comparison to that of a pre-stack common-offset Kirchhoff migration and (b) a subsequent true amplitude zero-offset migration needs only a simple weight (Hubral et al., 1991; Schleicher et al., 1993). Moreover, the stack of all obtained simulated zero-offset sections will lead to a better stacked section than the conventional process of constant-velocity NMO/DMO.

MZO, although not necessarily true-amplitude MZO, may also be considered for velocity analysis. Because of the limited spatial extent of the MZO stacking curve, MZO image gathers, which are often also called DMO gathers, are cheaper to produce than conventional pre-stack-migration image gathers. They are, however, of similar practical value to estimate velocity errors.

CONCLUSION

In this paper, we have formulated an approach to a true-amplitude migration to zero offset (MZO) for 2.5-D in-plane reflections in 2-D laterally inhomogeneous media with curved interfaces. Constructing true MZO amplitudes (in our sense) implies that in the simulated zero-offset reflections the original geometrical-spreading factor of the common-offset reflections is replaced by that of corresponding actual zero-offset reflections for the same reflection points. This goal is achieved by a weighted one-fold single-stack integral in the time domain along specific stacking curves. These turn out to coincide with the MZO inplanats as defined by Hubral et al. (1996). Not relying on any property of the arbitrarily curved reflectors to be imaged, the true amplitude weight function can be computed by dynamic ray tracing performed along ray segments that link the zero-offset and common-offset source and receiver points to certain points in the macro-velocity model. The procedure closely parallels that of a pre-stack Kirchhoff-type common-offset migration, where the rays from the sources to the receivers are connected by “diffraction points.” It therefore makes sense to refer to the proposed MZO approach as a Kirchhoff-type, or shortly, a Kirchhoff MZO.

Using standard asymptotic considerations, our analysis has shown how, irrespective of the chosen weight, any single-stack MZO affects the amplitude of the resulting simulated zero-offset reflections. The MZO output is seen to be proportional to the zero-offset geometrical-spreading factor, while the proportionality factor is dependent on the reflector overburden only and not on the reflector dip and curvature. This important result was crucial to the formulation of a true amplitude MZO weight function valid for curved reflectors in inhomogeneous media. Note also that the proposed general true amplitude Kirchhoff MZO for inhomogeneous media may not only be of use for the general 2.5-D situation described here, but it may also help to find analytic expressions for stacking curves and weight functions in simpler types of media, e.g., for a velocity distribution that varies with depth only.

Analyzing the result of a Kirchhoff MZO in the vicinity of the desired zero-offset reflection time, we found a simple and geometrically appealing formula for the pulse stretch seismic reflections are subjected to when migrated from common-offset to zero offset. It turned out that the factor describing the pulse stretch is just the cosine of the reflection angle of the original common-offset reflections. Although this expression for the pulse stretch was obtained from studying Kirchhoff MZO, it is common to any other MZO method that is kinematically equivalent to the proposed approach.

The discussed theoretical features of 2.5-D true-amplitude Kirchhoff MZO have been demonstrated with the help of two simple numerical examples. We have seen that the amplitude recovery of a true-amplitude MZO is quite good. Note, however, that portions that are insufficiently illuminated by the original common-offset experiments cannot be expected to be correctly recovered in the simulated zero-offset sections.

Being designed a priori for 2-D laterally inhomogeneous velocity models, our approach differs from all MZO or DMO schemes proposed so far in the literature. To enable a comparison, we tailored our general formulas to the constant-velocity case. For the purpose of comparing different MZO and DMO integral methods, we have also shown how a two-fold smear-stack DMO integral is related to a one-fold single-stack Kirchhoff MZO or DMO integral. We have seen that for a constant velocity, the proposed method is kinematically equivalent to the known schemes. The constant-velocity form of the weight function derived in this work differs somewhat. This is due to the slightly different conceptions that other authors have of what a true amplitude MZO should achieve. The closest relatives to the present MZO weight are the ones of Bleistein (1990) and Black et al. (1993). Also Bleistein and Cohen (1995), Goldin and Fomel (1995), and Fomel and Bleistein (1996) have similar results for a curved reflector overlain by a constant-velocity medium.

We stress once more that this work generalizes the previous constant-velocity results to laterally inhomogeneous media, which has been so far an open problem. We were able to provide a substantial and natural extension of previously proposed schemes. We finally remark that the present Kirchhoff MZO was obtained along the same lines as the fairly general theory presented in Hubral et al. (1996) and Tygel et al. (1996), tailoring it to the specific problem of a 2.5-D true amplitude MZO. Other true-amplitude imaging problems, including other configuration transforms, may be tackled in a corresponding manner.

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PUBLICATIONS

Detailed results are accepted for publication in Geophysics (Tygel et al., 1998).