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Traveltime Multiparameter Estimation Using Bound Constrained Optimization: A First Approach

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ABSTRACT

For a fixed, central ray in anisotropic elastic or acoustic media, traveltime moveouts of corresponding rays in its viciniy can be well described in terms of a certain number of parameters that refer to the central ray only. The determination of these parameters out of multicoverage data leads to very powerful stacking algorithms to produce, e.g., simulated zero-offset sections, very often with better quality as the ones obtained by conventional common mid-point stack. Assuming two-dimensional propagation, so that sources and receivers are distributed on a single seismic line, the multiparametric traveltime expressions depend on three parameters. By a combination of just introduced spectral projected gradients and global optimization methods, a new algorithm has been obtained to directly extract the traveltime parameters out of coherency analysis applied directly on the data. Numerical results obtained in synthetic examples show an excellent performance of the method, both in accuracy and computational effort. The results obtained so far indicate that the algorithm may be a feasible option to solve the corresponding, harder, full three-dimensional problem.

INTRODUCTION

In the framework of zero-order ray theory, traveltimes of rays in the vicinity (paraxial) of a fixed (central) ray can be described by a certain number of parameters which refer only to the central ray. The approximations are correct up to the second order of the distances between the paraxial and central ray at the corresponding initial and end points. They are, thus, valid independently of any seismic configuration.

Assuming the central ray to be the primary zero-offset or normal reflection ray, the number of parameters (emergence angles and curvatures of certain wavefronts) are three and eight, for two- and thre-dimensional propagation, respectively. Determination of these parameters by coherency analysis directly applied to multicoverage

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data leads to very powerful stacking algorithms to produce, e.g., simulated zero-offset sections of superior quality.

For two-dimensional propagation, in which sources and receivers are distributed on a single seismic line, simple multiparametric traveltime expressions are available. The parameters are the emergence angle of the normal ray and the wavefront curvatures of the normal and normal-incident-point waves. All these quantities are defined at the central point, where the normal, primary reflection ray hits the measurement surface. For a given selection of the three parameters that refer to a fixed normal ray, each of the traveltime expressions explicitly describes the moveout of any reflection ray in terms of its arbitrary source and receiver locations with respect to the central point of the normal ray. Assuming different geometries of acquisition, the three-parameter formula can be reduced to different ones depending on two or one parameter.

To actually find the traveltime parameters by means of coherency analysis applied to the multicoverage data in a a stable and feasible way is the crucial problem that has to be solved to enable the application of the multiparameter stacking. In this work we present a new method, based on global optimization techniques, for the estimation of the parameters for two-dimensional multicoverage data. It uses a recently introduced nonmonotone, spectral projected gradient (SPG) optimization algorithm (Birgin et al., 1997). The method uses classical projected gradient together with nonmonotone line-search and spectral-steplength. The method is applied to find local maxima of a multiparametric semblance function. A further combination with global optimization techniques is then applied to obtain the corresponding global maximum.

Although the proposed strategy can be applied to find the three parameters simultaneously, we present in this paper only the results for the application of the method to the data originated by a particular arrangement of sources and receivers, namely the common shot configuration. In this situation, the number of parameters is reduced to two, which simplifies the search for the global maximum.

Note that the full solution can be obtained if another experiment is carried out. Using the data originated from the common mid-point configuration, we find the optimal value of a single quantitie, which is a combination of the original parameters. Collecting the results coming from both configurations, it is possible to find the optimal values of the three parameters.

HYPERBOLIC TRAVELTIME EXPANSION

We assume the subsurface to be described by a 2-D laterally inhomogeneous isotropic layered earth model, for which body-wave traveltimes can be adequately approximated by zero-order ray theory (see, e.g., Cervený, 1985). We suppose that a dense multicoverage seismic experiment has been carried out on a single seismic line along a horizontal x-axis. This implies that each point of the seismic line is surrounded by a set of shot-receiver pairs (within a certain range of offsets). In practice, this implicitly requires some trace interpolation to replace eventual missing traces.

As shown in Figure 1, we consider a fixed target reflector Σ in depth, as well as a fixed *central* point X_0 on the seismic line, considered to be the location of a coincident source- and -receiver pair $S_0 = G_0 = X_0$. Also shown in Figure 1 is the two-way normal, zero-offset reflection ray, $X_0 R_0 X_)$, called from now on the *central ray*. It hits the reflector at point R_0, known as the normal-incident-point (NIP). For a source- and receiver pair (S, G) in the vicinity of the central point, we consider the primary reflected ray SRG relative to the same reflector Σ. We use the horizontal coordinates x_0, x_S and x_G to specify the location of the central point X_0, the source S and the receiver G. It is convenient to introduce the midpoint and half-offset coordinates

$$x_m = \frac{x_G + x_S}{2} - x_0 \quad \text{and} \quad h = \frac{x_G - x_S}{2} \tag{1}$$

The simplest traveltime approximation for a two-dimensional primary reflected ray in the vicinity of a zero-offset, central ray, is probably the classical *hyperbolic* traveltime (see, e.g., Cervený, 1985; Ursin, 1982; Schleicher et al., 1993). For our purposes, we adopt the hyperbolic traveltime expression as described in Tygel et al. (1997), namely

$$T^2(x_m, h, \beta_0, K_N, K_{NIP}) = \left(t_0 + \frac{2x_m \sin\beta_0}{v_0}\right)^2 + \frac{2t_0 \cos^2\beta_0}{v_0} \left(K_N\, x_m^2 + K_{NIP}\, h^2\right) , \tag{2}$$

where t_0 is the zero-offset traveltime and β_0 is the emergence angle the zero-offset ray make with the surface normal at the central point (see Figure 1). The quantities K_N and K_{NIP} are the wavefront curvatures of the *normal* N- and *normal-incident-point* NIP-waves, respectively, both measured at the central point.

The N- and NIP- waves are fictitious eigenwaves introduced by Hubral (1983) for the analysis of the actual propagation of the zero-offset ray, as well as for its corresponding paraxial rays. Their wavefront curvatures at the central point carry important information about the velocity model in which the wave propagation takes place. The N-wave can be conceptually visualized such that its wavefront at zero time coincides with the reflector, and travels to the surface with half the medium velocity. It arrives at the central point at the same time as the zero-offset ray. The NIP-wave can be visualized as starting as a point source at the reflection point (R_0) of the zero-offset reflection ray and progresses upwards with half velocity of the medium. It also arrives at the central point at the same time as the zero-offset ray.

FORMULATION OF THE PROBLEM

The data obtained by a multicoverage seismic experiment performed on the seismic line consists of a multitude of seismic traces $U(x_m, h, t)$ corresponding to source-receiver pairs located by varying coordinate pairs (x_m, h) and recording time $0 < t < T$. The basic problem we have to solve is the following:

> Consider a dense grid of points (x_0, t_0), where x_0 locates a central point X_0 on the seismic line and t_0 is the zero-offset traveltime. For each central point X_0 let the medium velocity $v_0 = v(x_0)$ be known. From the given multicoverage data, determine, for any given point (x_0, t_0) and velocity v_0, the corresponding parameters β_0, K_N and K_{NIP}.

The general approach to solve this problem is to apply a multiparameter coherency analysis (semblance) to the data, using the traveltime formula (2) to a number of traces $U(x_m, h, t)$ in the vicinity of the central ray X_0 and for a suitable time window around the time t_0. The desired values of sought-for parameters will be the ones for which one achieves maximum coherence when applying the traveltime (2) to the data.

From the above considerations, it should be expected, right from the start, that points (x_0, t_0) in the vicinity of a zero-offset primary reflection arrival should produce, for the correct parameters, a large coherency measure. On the other hand, one is expected to find small coherency at points where no such arrival occurs. Assigning at each point (x_0, t_0) its corresponding semblance, one obtains a seismic section *semblancegram* (Gelchinsky at al., 1987), which can be seen as a simulated or stacked zero-offset section that pertains to the multicoverage data. In the same way, similar sections can be obtained using the parameters β_0, K_N and K_{NIP}, respectively.

OPTIMIZATION TECHNIQUE

For each pair (x_0, t_0) the objective is to find the global maximum of the *semblance* function, which depends on the parameters β_0, K_N and K_{NIP}. These parameters are restricted to the ranges $-\pi/2 < \beta_0 < \pi/2$ and $-\infty < K_N, K_{NIP} < \infty$. For simplicity, we shall omit the dependency on x_0 and t_0 in all functions described below.

Given the seismic traces $U(x_m, h, t)$, and the vector of parameters $P = (\beta_0, K_N, K_{NIP})$, the semblance function S is given by

$$S = \frac{[\sum U(x_m, h, T(P))]^2}{M \sum [U(x_m, h, T(P))]^2} \tag{3}$$

where $T(x_m, h, P)$ is given by equation (2), M is the total number of traces, and the summing is performed over all traces.

In order to obtain a differentiable function which allows us to use a gradient-type method for the optimization of the semblance function S, we apply a differentiable interpolation strategy, namely B-splines, for the computation of $U(x_m, h, t)$ at $t = T(P)$.

The strategy for computing the global maximum of the semblance function is as follows. We begin by running a local method, the *Spectral Projected Gradient* (SPG) method from different initial points. As usual, these type of methods guarantee convergence to stationary (minima, maxima or saddle) points only. For this reason, we define a mesh of initial points and take as the global maximum, the obtained solution with maximum value.

The SPG method takes in each iteration the spectral scaled projected gradient as a feasible ascent direction, and performes a nonmonotone line search to guarantee sufficient increase of the semblance function. The idea of the nonmonotone strategy is to look for a point which gives a greater function value with respect to the last ten function values obtained. The main advantage of this method is that it uses first-order (gradients) information only, which reduces both the computational efforts and storage requirements. For complete details about the SPG method, we refer the reader to Birgin et al. (1997).

COMMON SHOT CONFIGURATION

As it is common practice in seismic processing, we will make use of a special *source-receiver gather*, namely particular arrangement of source-receiver pairs, according to certain pre-assigned configuration. In the *Common-Shot* configuration (CS), the central point is a fixed source and the receivers vary. The location of the source-receiver pairs in the CS configuration are specified in midpoint and half-offset coordinates by the condition $x_m = h$. Substituting into the traveltime formula (2) yields the CS traveltime formula

$$t_{CS}^2(h) = \left(t_0 + \frac{2h\sin\beta_0}{v_0}\right)^2 + \frac{2h^2 t_0 \cos^2\beta_0}{v_0}\left(K_N + K_{NIP}\right) . \qquad (4)$$

By restricting the traces to conform to the particular arrangement of sources and receivers, the CS configuration, one can accordingly reduce the number of parameters in the corresponding traveltime expression. In the CS configuration, we easily see that the search is restricted to the two parameters β_0 and $K_N + K_{NIP}$ (see formula (4)).

The strategy of using particular configurations to reduce the number of parameters to be estimated has advantages and disadvantages. The main advantage is the sometimes significant reduction of computational effort. As a disadvantage, less redundancy is made use for, as many traces that do not conform to the selected configuration have to left out.

In the next section, we will make use of the CS traveltimes as a strategy to derive the parameters β_0 and $K_N + K_{NIP}$ from the multicoverage data.

NUMERICAL RESULTS

We consider the model of a single smooth reflector between two homogeneous half-spaces (see Figure 2). The constant velocities above and below the reflector are v_1 =2.5 km/s and v_2 =2.6 km/s, respectively. The input data for our experiment is a colection of 61 common-shot seismic sections with 30 traces each one, where the source (x_0) is in the range from 0 km to 0.6 km and the time window is [0.4s,9.11s]. To simulate real situations we have added a colored noise of 20%. This was obtained uppon the convolution of white noise with the wavelet used to construct the seismograms. Figure 3 shows the corresponding seismic section for the case x_0 =0.35 km.

Figure 4 shows the maximum semblance function value obtained for each pair (x_0, t_0). These values were computed using the global optimization technique described in the previous sections. Note the very good agreement with the real zero-offset seismic section depicted in Figure 5, generated by forward modeling.

All the experiments were run in an ORIGIN2000, with FOUR processors R1000 of 195MHz, 4MB of cache memory, and 1024MB of RAM memory. We used the language C++ with the MIPSpro Compilers: version 7.20 and the optimization compiler option -Ofast. In spite of the great number of runnings of the SPG algorithm ($\approx 2.10^6$), the total CPU time used under the mentioned computer environment did not exceed 20 minutes.

FINAL REMARKS

The results obtained here are encouraging. We have made a first attempt to apply optimization techniques to the traveltime multiparameter estimation. Experiments for the estimation of all parameters without using particular configurations are needed to confirm the efficiency of our strategy. This is subject of current research.

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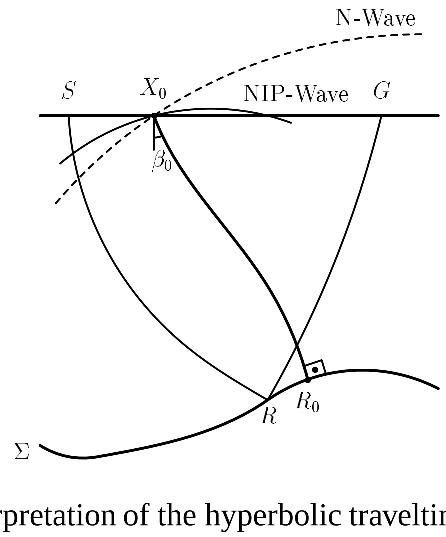

Figure 1: Physical interpretation of the hyperbolic traveltime formula parameters.

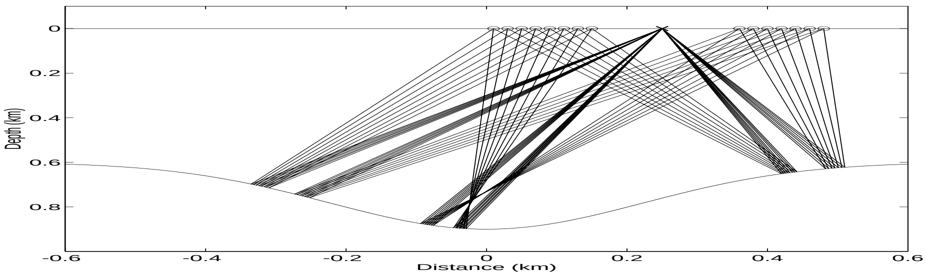

Figure 2: Model and acquisition geometry.

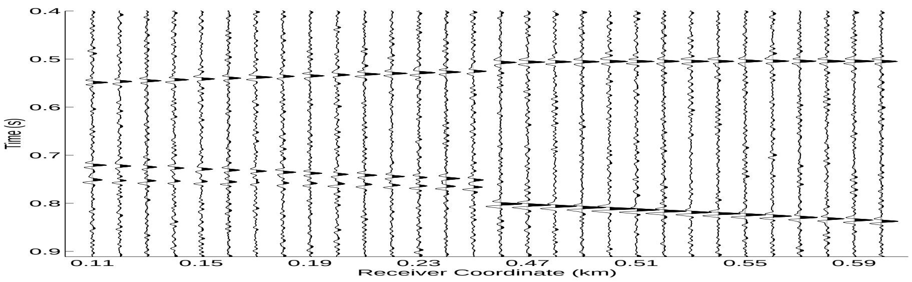

Figure 3: Seismic section with 20% of colored noise.

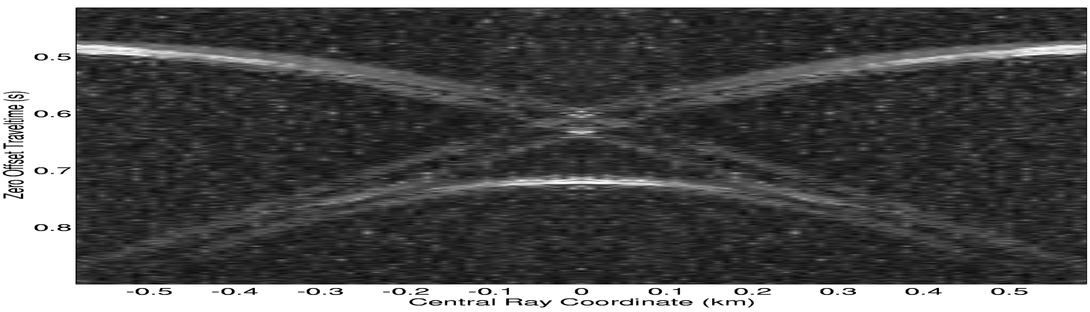

Figure 4: Semblancegram.

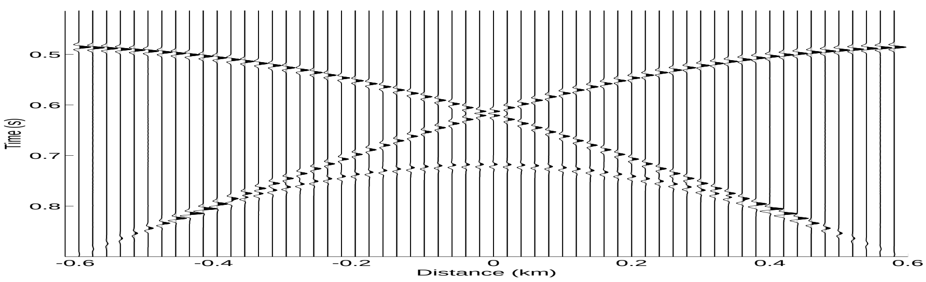

Figure 5: Simulated zero-offset section.