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Green's Function computation with recursive cell tracing

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ABSTRACT

A recursive cell wave front tracing algorithm provides us with a method for generating interpretable wave phases in 3D earth models. We developed the algorithm further, such that it is capable of incorporating source radiation and receiver radiation patterns, which are important ingredients for elastic modeling and imaging. Applications of this algorithm include the computation of Green's Function and subsequent generation of synthetic prestack data sets. A parallel MPI based implementation is in progress.

INTRODUCTION

The recursive cell wave front tracing algorithm, as described by Moser Moser and Pachel (1997), provides us with a method for generating interpretable wave phases in 3D earth models. Using travel times and amplitudes in a smooth background model, we are able to use this information for purposes of imaging as well as modeling. We are able to compute Greens functions in complex subsurface models. We developed the method further, such that it is capable of incorporating source radiation and receiver radiation patterns, which are important ingredients for elastic modeling and imaging. Using the SEG/EAGE salt model we generate primary p-wave Green's functions from various target areas. Using the computed travel time amplitude and phase information, we synthesize entire prestack data sets. Thus we are able to to compare prestack data sets that were modelled using full wave form techniques. Due to the potentially heavy computational load the methods is well suited to analyses target oriented primary events. Such scenarios represent ideal test cases for multiple suppression techniques and target-oriented imaging comparisons.

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APPLICATION TO THE SEG/EAGE SALT MODEL

We demonstrate the ability to compute complex subsurface responses by applying it wave front tracking method to the SEG/EAGE salt model. This model had been use previously (Karrenbach, 1998) to compute full wave form responses acoustically and elastically using a Finite Difference method. The complexity of the seismic response in this still simple model is not easily analyzed. Using this wave front tracking method we can compute targeted responses that are ray-theoretically correct. Figure 1 shows the 2D slice that has been used to compute various responses. In this example we consider p wave types only. Figure 2 shows the resulting zero offset section, which contains only the p wave responses from the top of the salt. This section has been generated by computing the Green's Function with source points at discrete grid locations at the top of the salt and receiver locations at the top surface of the model.

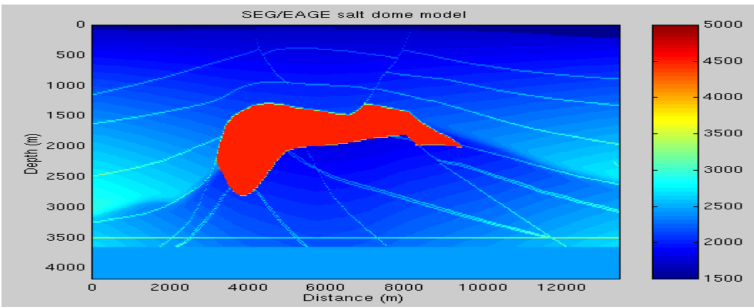


Figure 1: Salt model 2D slice.

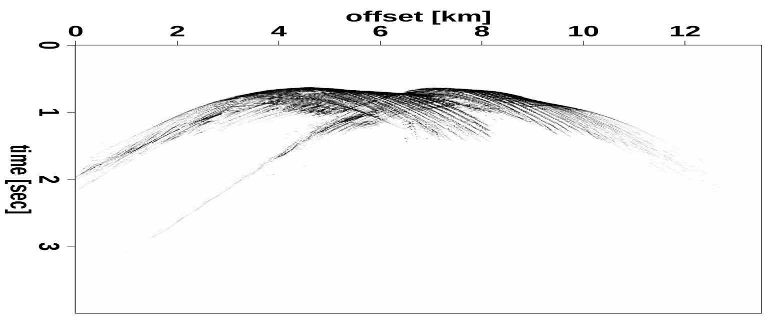


Figure 2: Zero offset section containing p wave events originating from top of the salt only.

Using the already computed Green's function information, we can generate an entire prestack data set. Figures 3 and 4 show the prestack data set at various time slices. The source location spacing was at a regular 40 meter interval. In contrast to the full wave form response we obtain a much simpler response that shows the events of waves, that emanated from the top of the salt and propagated in the complex overburden up to the receivers. As starting angles only a cone of width +/- 90 degree from the vertical was allowed. Still many of the events are generated by multi-pathing rays, that get refracted at the corners of the salt and at the upper interfaces.

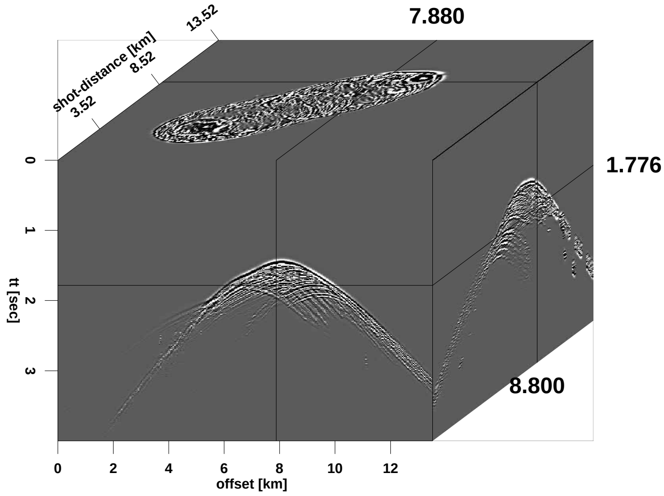


Figure 3: Shotpoints are located every 40 meters. We obtain a prestack data set. The times lice is placed at 1.776 seconds.

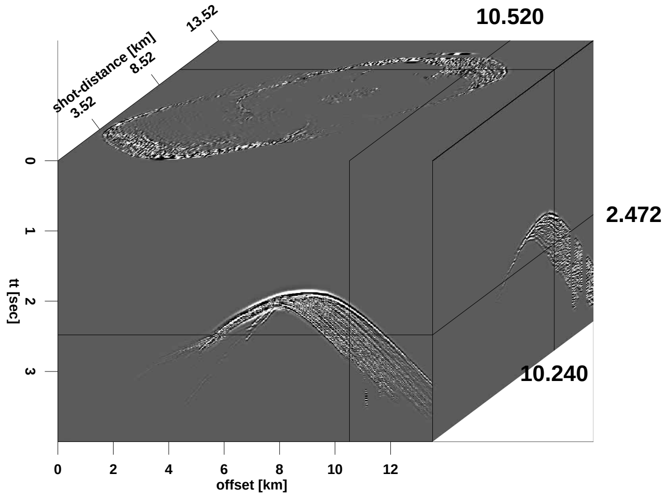


Figure 4: Shotpoints are located every 40 meters. We obtain a prestack data set. The times lice is placed at 2.472 seconds.

In contrast to the partial wave field response in the previous figures, Figure 5 shows the p-wave response obtained by including all subsurface scatterers. Reaching already a complexity similar to the one obtained by full wave field modeling techniques.

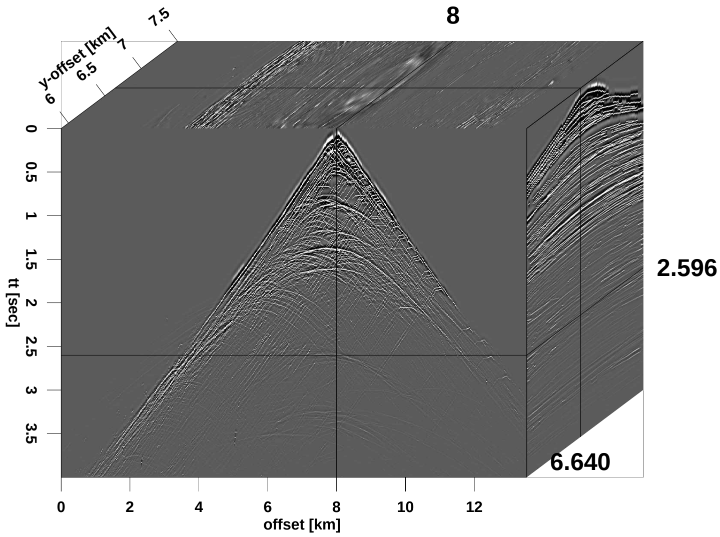


Figure 5: Snapshot from seismogram with shot point at 8 km offset at 2.59 seconds (filtered with a bandpass filter 10 -80 Hz).

The previous comparison all showed the application of the standard scalar wave front tracing scheme. If we would like to compute a elastic response of the subsurface, that is recorded at multi component receivers at the surface we need to augment the algorithm to keep proper track of particle motions along a wave front path.

In the next section we briefly outline our scheme to propagate polarization vectors through the medium using the recursive cell ray tracing.

SHEAR WAVE PARTICLE MOTIONS

The generic recursive cell ray tracing algorithm is based on Moser's method **?**), which solves the ray equations in the following form

$$\boldsymbol{T} = \frac{d\boldsymbol{x}}{dt} = u(x)^{-2}\boldsymbol{p} \tag{1}$$

$$\frac{d\boldsymbol{p}}{dt} = u(x)^{-1} \bigtriangledown u(x) \tag{2}$$

where $u(x)$ is the slowness at **x**, $u = \frac{1}{c}$, $\boldsymbol{p}$= the slowness vector, $\bigtriangledown$u(x) is the gradient of the slowness at x and $\boldsymbol{T}$ is the tangent vector to the ray. This system of equations is solved numerically by means of a Runge-Kutta scheme. Cartesian coordinates are used throughout this formulation. In its original implementation calculations for the displacement vector are neglected since only single p-wave type were considered. The p-wave polarization can be deduced directly from computed quantities, because its displacement vector is tangent to the ray. In order to develop an algorithm which is suitable for computing s-waves and their displacement vector Moser's algorithm needs to be modified to use a source radiation pattern. The initial polarization vector $\boldsymbol{e}_j$ at the source defines a characteristic source strength for this radiation angle.

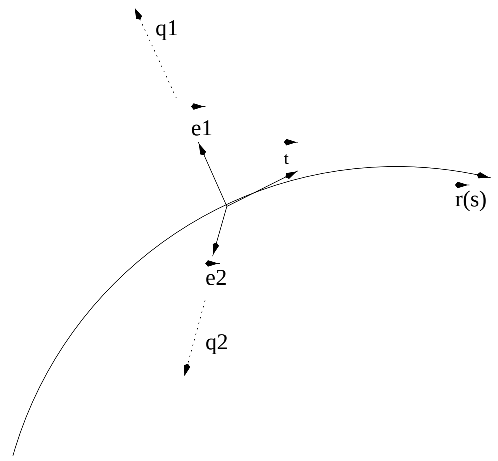


Figure 6: A ray and its unit vectors/polarization vectors, after Popov and Psencik (1976)

The displacement vectors are described by two perpendicular unit vectors $\boldsymbol{e}_j$, with ranging from $j = 1, 2$:

$$\frac{d\boldsymbol{e}_j}{ds} = \chi_j \boldsymbol{T} \tag{3}$$

$$\chi_j = \frac{1}{c}\frac{\partial c}{\partial q_j} \mid_{q_j=0} \tag{4}$$

where c is velocity, and $\boldsymbol{T}$ is the tangent vector and q_j are the ray centered coordinates in the $\boldsymbol{T}, \boldsymbol{e_1}, \boldsymbol{e_2}$ system. $\boldsymbol{e}_j$ and $\boldsymbol{T}$ are in unit length. We can solve this integral by a finite difference approximation

$$\boldsymbol{e}_j(0 + \triangle s) = \chi_j \boldsymbol{T}(0) \triangle s + \boldsymbol{e}_{j(0)} \tag{5}$$

where q =ray centered coordinates and $\triangle s$ = step size for the calculations along the ray.

We use Fermat's integral in the following form

$$F = \int \frac{\sqrt{x'^2 + y'^2 + z'^2}}{c(x, y, z)} d\sigma, \tag{6}$$

where c is the velocity and $\sqrt{x'^2 + y'^2 + z'^2} d\sigma = ds$ is the arc length along a ray $\boldsymbol{r} = \boldsymbol{r}(s) = (x(s)\boldsymbol{i} + y(s)\boldsymbol{j} + z(s)\boldsymbol{k})$.

Subsequently its Euler's equation take the form

$$\frac{d}{ds}\left(\frac{1}{c}\frac{d\boldsymbol{r}}{ds}\right) - \bigtriangledown\frac{1}{c} = 0. \tag{7}$$

Taking the time t as a parameter along the ray instead of s then and using a slowness vector equation 7 takes on the form of a system of two equations

$$\Rightarrow \frac{d\boldsymbol{r}}{dt} = c^2\boldsymbol{p} = u^{-2}\boldsymbol{p} \tag{8}$$

$$\frac{d\boldsymbol{p}}{dt} = u^{-1} \bigtriangledown u. \tag{9}$$

This system is solved numerically and for a practical implementation we consider the computation of the polarization vector in Cartesian coordinates. In the following $(,)$ denote inner products. From Fermat's integral

$$f(x, y, z) = f(x(s), y(s), z(s)) = F(s) \tag{10}$$

$$\frac{dF}{ds} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds} + \frac{\partial f}{\partial z}\frac{dz}{ds} = (\bigtriangledown f, \boldsymbol{T}) \tag{11}$$

we obtain a different formulation for equation 4

$$\chi_j = -\frac{1}{u}(\bigtriangledown u, \boldsymbol{e}_j), \qquad j = 1, 2 \tag{12}$$

where $(\bigtriangledown u, \boldsymbol{e}_j)$ denotes a inner product between the gradient of the slowness and the jth polarization vector.

Finally, approximating

$$\frac{d\boldsymbol{e}_j}{dt} = \frac{1}{u}\chi_j\boldsymbol{T} \tag{13}$$

by a finite difference quotient

$$\frac{d\boldsymbol{e}_j}{dt} = \frac{\boldsymbol{e}_{j+1} - \boldsymbol{e}_j}{\triangle t} = \frac{1}{u}\chi_j\boldsymbol{T} \tag{14}$$

we obtain the discretized equation

$$\boldsymbol{e}_{j+1} = \chi_j\boldsymbol{T}dt + \boldsymbol{e}_j \tag{15}$$

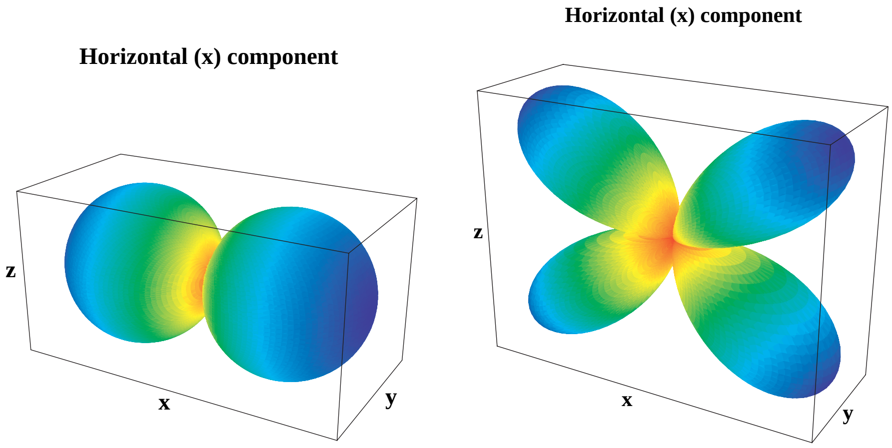
where dt =step is the step size for the numerical calculation.

Thus we can calculate the vectors $\boldsymbol{e}_{j+1}$ and $\boldsymbol{T}$ at the point given after a certain time increment dt from the source. Equation 15 is then solved for all 3 components. In fact we perform the computation by a Runge-Kutta scheme. In isotropic media we only need to track to independent polarization vectors namely $\boldsymbol{e}_1$ and $\boldsymbol{T}$. The second polarization vector $\boldsymbol{e}_2$ is obtained by computing the cross product $\boldsymbol{e}_2 = \boldsymbol{e}_1x\boldsymbol{T}$.

SOURCE AND RECEIVER CHARACTERISTICS

When modeling multi component seismic data one needs to pay particular attention to the source and receiver characteristics. Multi component sources can display complicated radiation patterns. A seismogram is then given by the combination the source and receiver radiation patterns as well as the propagation effects through the medium.

We augmented the original method to handle exactly this kind of additional complexity. These patterns describe the distribution of the amplitudes on a spherical wave front. We are able to apply any desired source radiation pattern. Typically we use the analytically given radiation field of a p, shear, dipole or double couple source as illustrated in Figure 7 and 8.



(a) (b)

Figure 7: Horizontal (x) component for P-wave energy due to a) explosion source b) double couple source.

CONCLUSION

We used Moser's original recursive cell ray tracing algorithm and augmented the method to include source and receiver characteristics as well as the tracking of polarization vectors through a medium. We show some application of this wave front tracking scheme by using the SEG/EAGE salt model, where we calculated partial and full single wave type responses. Green's Functions as well as synthetic prestack data are computed. The method is attractive since it uses a relatively low amount of memory and exhibits a natural amount or computational parallelism. Currently a Message Passing Implementation is being investigated.

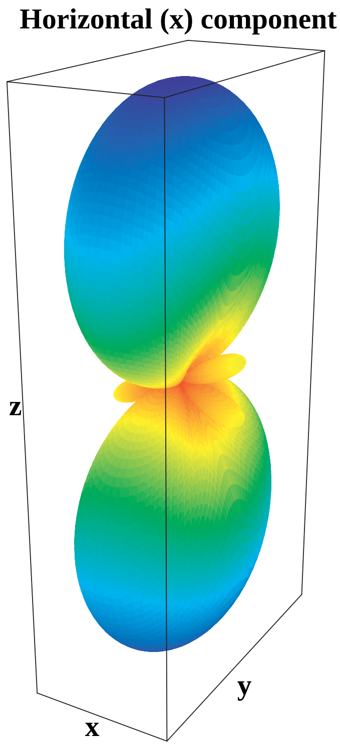


Figure 8: The horizontal (x) component of the radiation pattern of the S-wave source.

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PUBLICATIONS

Detailed results are being published in a Diploma Thesis by B. Traub.