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Dynamic equivalent medium approach for 2-D and 3-D random media

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ABSTRACT

Applying well-known techniques from scattering theory, we present a description of the wavefield of an initial plane wave vertically propagating in 2-D and 3-D random media. This proceeding enables us to construct the Green's function dependent on the second order statistics of the medium heterogeneities. In a further step we are going to obtain a valuable tool for inversion and modeling. We briefly show how to derive the statistical approximations and discuss their limitations.

INTRODUCTION

Seismic waves propagating in randomly multi-layered media are subjected to stratigraphic filtering. The physical reason is the multiple scattering by 1-D inhomogeneities. In random stationary media, explicit approximations for the transmissivities of obliquely incident P- and SV-plane waves have been found by applying the second-order Rytov approximation to the 1-D multiple scattering problem in the frequency domain. This description is known as the generalized O'Doherty-Anstey formalism (Shapiro et al. (1996a)). However, the earth, the lithosphere and especially reservoirs may have a very complex structure. Therefore the concept of 2-D or 3-D random media can be a more suitable and more general description. The random medium consists of a constant background of a certain medium parameter and its corresponding fluctuations with a given spatial correlation function. Amplitudes and phases of wavefields fluctuate in random media. Furthermore, averaged wavefields are characterized by attenuation and dispersion. The underlying motivation of understanding these wavefield characteristics is the construction of Green's functions for heterogeneous media. The latter is extremely useful in order to apply any kind of inversion technique, a fundamental problem in exploration seismics. This study is based on two theoretical studies of propagation of a compressional plane wavefield in 2-D or 3-D random media characterized by isotropic stationary velocity fluctuations (see Shapiro and Kneib (1993)

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and Shapiro et al. (1996b)). Due to scattering by the inhomogeneities the wavefield becomes distorted and can be described as a sum of coherent and incoherent wavefield. Which part of the wavefield is actually measured in experiments depends on the size of the used receiver. In geophysical applications the receivers are small compared with the wavelength and the size of inhomogeneities, such that both parts of the wavefield become significant. Therefore there may occur a discrepancy between the recorded wavefield and the coherent wavefield (meanfield). The latter is described by the so-called meanfield theory. Here we present a description of the wavefield in a perturbation approximation.

ATTENUATION AND DISPERSION

The paper of Shapiro and Kneib (1993) deals with scattering attenuation. Starting point for an analysis of acoustic scattering is the Lippmann-Schwinger equation, which is a integral solution to the scattering problem. Unfortunately, this is not an analytical closed representation and hence reduces its applications. One way to get around this problem is the linearization by a smooth perturbation approximation, often referred to as Rytov approximation (Ishimaru (1978)). The Rytov method describes the wavefield $u(\boldsymbol{r})$ as $u(\boldsymbol{r}) = \exp(\Psi(\boldsymbol{r}))$ and develops a series solution of the exponent $\Psi(\boldsymbol{r})$. The real part of $\Psi(\boldsymbol{r})$ represents the fluctuations of the logarithm of the amplitude (log-amplitude fluctuations); the imaginary part represents phase fluctuations. For not too long travel distances, where the coherent part of the wavefield dominates, analytical expressions are found for the mean logarithm of amplitude in the first Rytov approximation. That is, relations are established between the autocorrelation functions of the velocity fluctuations and the autocorrelation functions of the amplitude fluctuations. The paper of Shapiro et al. (1996b) follows along the same strategy: the averaged phase can be described by the crosscorrelation functions of amplitude level and phase fluctuations as well as the coherent phase obtained in the Bourret approximation which is in fact an approximation to the Dyson equation (Rytov et al. (1987)). The phase fluctuations cause traveltime fluctuations when considering wavefield registrations at points of surfaces parallel to the wavefront of the initial plane wave. This results in the so-called velocity shift and is physically explained by the fast-path effect (see e.g. Samuelides and Mukerji (1998)). The validity range of these descriptions are weak wavefield fluctuations and inhomogeneities with spatial sizes of the order or larger than the wavelength. More precisely, weak fluctuations mean that the log-amplitude variance is smaller than 0.5.

GREEN'S FUNCTION

We consider a time-harmonic plane wave vertically propagating in a 2-dimensional random medium. The next step is to combine the results of the mean logarithm of amplitude and phase fluctuations, as discussed in the previous section, in order to get a Green's function of the medium as a function of travel-distance, variance of velocity fluctuations, correlation length of the assumed correlation function and frequency, respectively. In the frequency domain we obtain the following results for the ensemble-averaged exponent $\langle\Psi(L, \sigma_v^2, a, k)\rangle$:

$$\langle\Psi(L, \sigma_v^2, a, k)\rangle = \langle\chi\rangle + i\langle\phi\rangle \tag{1}$$

with

$$\langle\chi\rangle = -\sigma_{\chi\chi}^2 \quad , \tag{2}$$

$$\langle\phi\rangle = \phi_c - \phi_0 - \sigma_{\chi\phi}^2 \tag{3}$$

where L means travel-distance in the vertical direction, σ_v^2 is the variance of the velocity fluctuations, a the spatial correlation length of the fluctuations and k is the background wavenumber ($k = \frac{\omega}{c}$, when ω denotes frequency and c the constant background velocity), respectively. The amplitude level variance and crossvariance of amplitude level and phase fluctuations read in the first Rytov approximation:

$$\sigma_{\chi\chi}^2 = 2\pi k^2 L \int_0^\infty d\xi \left(1 - \frac{\sin(\xi^2 L/k)}{\xi^2 L/k}\right) \Phi^{2D}(\xi) \tag{4}$$

$$\sigma_{\chi\phi}^2 = 4\pi k^3 \int_0^\infty d\xi \frac{\sin^2(\xi^2 L/2k)}{\xi^2} \Phi^{2D}(\xi). \tag{5}$$

Finally, the coherent phase can be written in the Bourret approximation as

$$\phi_c - \phi_0 = 4\pi k^3 L \int_{2k}^\infty \frac{\Phi^{2D}(\xi)}{\sqrt{\xi^2 - 4k^2}}. \tag{6}$$

The function $\Phi^{2D}(\xi)$ is the 2-D Fourier transform of the autocorrelation function of the velocity fluctuations. The corresponding results for 3-D random media read similar. These are not analytical closed recipes, but they can be easily evaluated by numerical integration. In the particular case of exponentially correlated velocity fluctuations, the above expressions can be simplified to give

$$\langle\chi\rangle = -L\sigma_v^2 a k^2 \left(1 + \frac{\pi}{2}\left[J_1(b)\cos(b) + Y_1(b)\sin(b)\right]\right) \tag{7}$$

$$\langle\phi\rangle = 2\sigma_v^2 a^2 k^3 \left(Lr\left[F(\frac{\pi}{2}, r) - E(\frac{\pi}{2}, r)\right] - \int_0^\infty d\xi \frac{\sin^2(\xi^2 L/2k)}{\xi^2(1 + \xi^2 a^2)^{\frac{3}{2}}}\right), \tag{8}$$

where

$$b = \frac{L}{2a^2k} \qquad r = \frac{1}{\sqrt{1+4a^2k^2}}. \tag{9}$$

Here J_1 and Y_1 are the Bessel functions of the first and second kind, F, E are the elliptic integrals of the first and second kind. In equation (8) only one integral remains to be solved numerically. These relations also hold for isotropic elastic random media. Gold (1997) discussed the generalizations of the Rytov as well as the Bourret approximations to elastic media. Therefore the results obtained for acoustic waves should be applicable in elastic media.

By means of the inverse Fourier transform, we obtain the time-dependent response due to the initial plane wave. Currently we are working on the confirmation of the analytical proceeding by finite difference simulations of wave propagation in 2-D elastic random media with Gaussian, exponential and von Karman correlation functions. An extension of these results to the case of anisotropic random media, characterized by anisotropic spatial correlation functions, should generally be possible. Such models are meaningful for investigations of wave propagation in the lithosphere as well as in seismic exploration.

CONCLUSION

We consider a time-harmonic plane wave traveling through a random medium which is assumed to be a realistic model of reservoirs and large regions of the lithosphere. These media are characterized by stationary velocity fluctuations as well as a spatial correlation function. With help of the Rytov approximation we obtain a description of the wavefield which takes into account the multiple forward scattering. Applying the inverse Fourier transform we get the wavefield due to a delta-pulse excitation. This is nothing else but the Green's function. With that knowledge arbitrary waveforms may be investigated. In other words, we are going to obtain the Green's functions of statistical macro models of small-scale heterogeneities. The results can be useful for modeling as well as inverse problems.

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