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Sensibility Analysis of the Multifocusing Traveltime Approximation

J. C. R. Cruz, P. Chira, and G. Garabito[1]

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ABSTRACT

The zero-offset (ZO) seismic section can be simulated by properly stacking a set of multi-offset seismic data, using conventional procedures like the well know Common-Midpoint (CMP/DMO) method. In the recent past years, a new stack technique for simulating a ZO section was proposed, the so-called Multifocusing (MF) Stack. This technique can be used for arbitrary configuration and number of source and geophone pairs.The multifocusing traveltime approximation of the stack formula depends on three wavefront parameters: (1) the radius of curvature of the NIP wave, R_{NIP} ; (2) the radius of curvature of the normal wave, R_N; and (3) the emergence angle of the reflection normal ray, β_o. These three wavefront parameters are obtained as solution of an inverse problem, in sense that they provide the best fitting of the stack surface on the observed multicoverage seismic reflection data. In this paper we present a sensibility analysis of the traveltime function, by analysing the first derivative of the multifocusing traveltime with respect to the searched-for wavefront parameters. This result is important to indicate the resolution power of the optimization procedure based on the multifocusing formulas.

INTRODUCTION

In Hubral (1983) the zero-offset geometrical spreading factor is described with help of two ficitious wave, the so-called Normal-Incidence-Point Wave (NIP) wave and the Normal Wave (N) wave. In recent works of Tygel et al. (1997) and Gelchinsky et al. (1997), we have seen that the same ficitious waves, NIP and N waves, can be also used to describe new paraxial traveltime approximations, that are useful for simulating zero-offset seismic sections. In this new approximations the traveltime in the paraxial vicinity of a central ray is described by certain number of parameters related with the central ray. If the central ray is the normal ray, and we assume a bi-dimensional wavefield propagation, they are three parameters: (1) The radius of curvature R_{NIP} ; (2)

[1]**email:** jcarlos@marajo.ufpa.br

the radius of curvature R_N ; and (3) the emergence angle β_o. The near surface velocity v_o is considered a priori known in the vicinity of the emergence point of the normal reflection ray. It is important to observe that there are several possibilities to express such traveltime approximation. To be known we have two second-order approximations, namely parabolic and hyperbolic given by Ursin (1982) and Tygel et al. (1997), and a double-square-root, also called multifocusing (MF) traveltime approximation, given by Gelchinsky et al. (1997). By using a hyperbolic approximation, Mueller et al. (1998) applied the so-called Common-Reflection-Surface (CRS) Stack method to synthetic seismic data, in a noise enviroment, by considering a heterogeneous layered medium. As result it was shown that this new technique is able to simulate zero-offset sections and as by-product gives the three wavefront parameters, which are useful for developing macrovelocity inversion procedures as found in Cruz and Martins (1998). More recently Landa et al. (1999) and Mueller (1999) have shown results of the MF and CRS stack methods,respectively, applied to real data. As part of the inverse problem, we describe the behavior of the MF stack surface, by analysing the first derivative of the MF traveltime approximation with respect to each one of the searched-for parameters, and also using the semblance function as coherence measure.

MULTIFOCUSING TRAVELTIME

In the Workshop on Macrovelocity Independent Imaging Methods taked place in Karlsruhe, Germany, 1999, three families of new seismic stack methods were presented: (1) POLYSTACK; (2) Multifocusing (MF) stack; and (3) Common Reflection Surface (CRS) stack. All of them does not use explicit information about the velocity model, and have as main subject to simulate zero-offset sections.

In this section we present the MF traveltime formula that was first given by Gelchinsky et al. (1997), and rewritten by Tygel et al. (1997) as

$$\tau = \tau_o + \Delta\tau_S + \Delta\tau_G \tag{1}$$

where

$$\Delta\tau_S = \frac{r_S}{v_o}\left[\sqrt{1 + 2\frac{\sin\beta_o}{r_S}\Delta x_S + \left(\frac{\Delta x_S}{r_S}\right)^2} - 1\right], \tag{2}$$

and

$$\Delta\tau_S = \frac{r_G}{v_o}\left[\sqrt{1 + 2\frac{\sin\beta_o}{r_G}\Delta x_G + \left(\frac{\Delta x_G}{r_G}\right)^2} - 1\right], \tag{3}$$

in which r_S and r_G are wavefront radii of curvature at the source and receiver, respectively. The source and receiver separations to the central point are $\Delta x_S = x_m - h - x_o$

and $\Delta x_G = x_m + h - x_o$. The corresponding wavefront curvatures are given by

$$K_S = \frac{K_N - \gamma K_{NIP}}{1-\gamma}, \qquad K_G = \frac{K_N + \gamma K_{NIP}}{1+\gamma}, \tag{4}$$

where

$$K_{NIP} = \frac{1}{R_{NIP}}, \qquad K_N = \frac{1}{R_N} \qquad \text{and} \qquad \gamma = \frac{h}{x_m - x_o} \tag{5}$$

In the equation (1) τ_o is the zero-offset reflection traveltime. x_o is the horizontal coordinate of the emergence point of the reflection normal ray, x_m and h are the midpoint coordinate and half-offset corresponding to a source-geophone pair. In this formula γ is the focus parameter defined by Gelchinsky et al. (1997). For a specified point $P_o(x_o, \tau_o)$ in the time section with the respective three wavefront parameters K_{NIP}, K_N and β_o we calculate the paraxial traveltimes, that define the surface trajectory used to stack the seismic data, Figure (1).

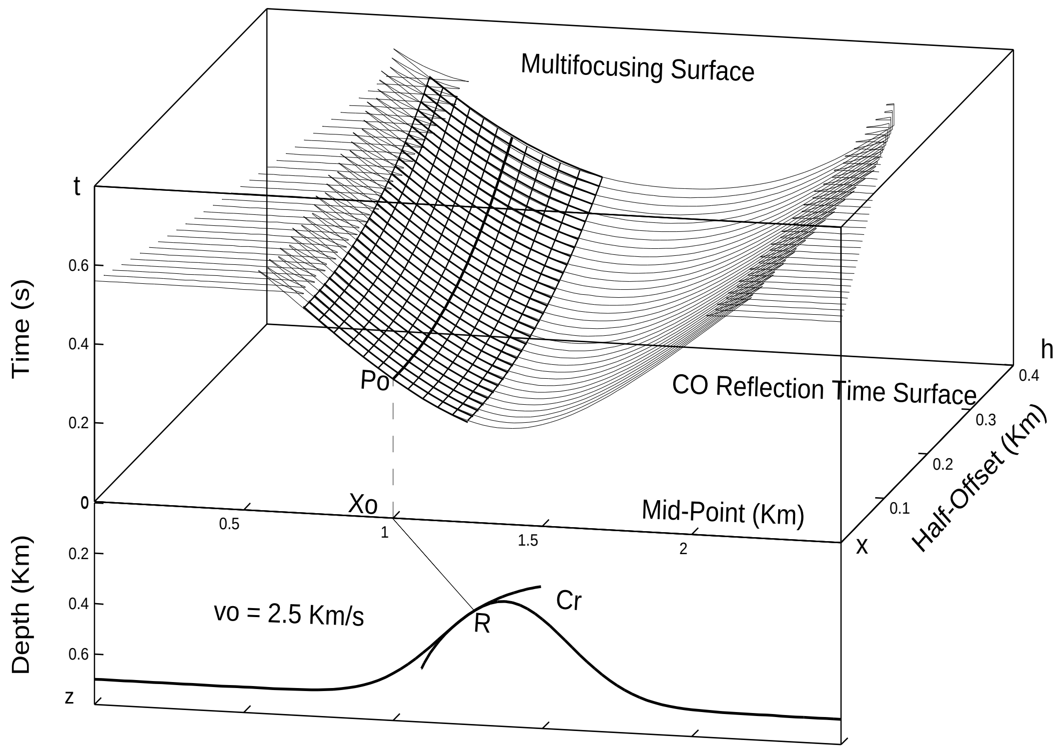

Figure 1: The traveltime curves for several half-offsets calculated by ray tracing and using the dome structure in the depth model. The corresponding MFS stack surface is lying on the ray theoretical traveltime curves.

INVERSE PROBLEM

The seismic imaging is successfully treated as an inverse problem, where the available data are the observations of the scattered wavefield and some incomplete information about the wavespeeds in the earth interior. The scattered wavefield, i.e. the reflector response, is considered to be governed by some system of wave equations, and the subject of the inverse problem is to determine the material parameters and their discontinuity surfaces.

In this paper we deal with other class of inverse problem, where the main goal is to simulate zero-offset sections by properly stacking of multichannel seismic data into an arbitrary configuration. To achieve the optimal stacking trajectory, a coherence measure is applied to the input data. The inverse problem is then put as finding the best stacking trajectory in sense of maximizing the chosen coherence criterium, the so-called coherence inverse problem.

The stacking trajectory corresponds to a surface built by a time function in the midpoint and half-offset domain. The time function corresponds to the multufocusing traveltime presented in the early section of this paper, and the coherence criterium is the semblance measure given by Neidell and Taner (1971),

$$S = \frac{\sum_{j=k-(N/2)}^{k+(N/2)} \{\sum_{i=1}^{M} f_{i,j(i)}\}^2}{M \sum_{j=k-(N/2)}^{k+(N/2)} \sum_{i=1}^{M} f_{i,j(i)}^2}, \tag{6}$$

where M is the number of channels, $(N+1)$ is the number of data sample into the time gate, $f_{i,j(i)}$ is the seismic signal amplitude indexed by the channel order number, $(i = 1, ..., M)$, and the stacking trajectory, $(j(i) = k - (N/2), ..., k + (N/2)$. k is the index of the amplitude in the center of the time gate.

The semblance function S means the ratio of signal energy to total energy, with values between $0 \leq S \leq 1$. The sample amplitude $f_{i,j(i)}$ is singled out from the multichannel data through the stack surface defined by the multifocusing traveltime approximation.

The coherence inverse problem is then formulated as determining the optimal vector of parameters $\mathbf{p} = [K_{NIP}\, K_N\, \beta_o]^T$, subjected to maximize the objective function (6) calculated from the stacking trajectory (1), by considering a fixed point $P_o(x_o, \tau_o)$.

It is of considerable interest to know how the MF traveltime is sensitive to variations in the searched-for parameter vector space. In the inverse theory this is what we call the sensibility analysis, what is the subject of the next section.

SENSIBILITY ANALYSIS

The most important step toward obtaining a simulated zero-offset section by the multifocusing traveltime approximation is the optimization procedure to find the trio (K_{NIP}, K_N, β_o). In general it is necessary to expend very much computational effort and time to find out which combination of parameters is the best one. It is a basic question for any optimization procedure, how sensitive is the functional that simulated the observed data to variations in the searched-for parameters. This question is answered here after analysis of the first derivative of the referred paraxial traveltime function (1) with respect to each one of the wavefront parameters. The three derivatives are given as follows (APPENDIX A)

$$\frac{\partial \tau}{\partial K_N} = \frac{\partial \Delta\tau_S}{\partial K_N} + \frac{\partial \Delta\tau_G}{\partial K_N} \tag{7}$$

$$\frac{\partial \tau}{\partial K_{NIP}} = \frac{\partial \Delta\tau_S}{\partial K_{NIP}} + \frac{\partial \Delta\tau_G}{\partial K_{NIP}} \tag{8}$$

$$\frac{\partial \tau}{\partial \beta_o} = \frac{\partial \Delta\tau_S}{\partial \beta_o} + \frac{\partial \Delta\tau_G}{\partial \beta_o} \tag{9}$$

These derivatives, equations (7), (8), and (9), are shown in the Figure 2 for half-offsets $0 < h < 1.2$ Km in midpoint domain. We remind that in this analysis we consider a fixed point $P_o(x_o, \tau_o)$ in the zero-offset seismic section, as shown in the Figure (1).

In the Figure (2) we have the multifocusing traveltime derivative is very high sensitive to variations of the parameters β_o and K_{NIP}, while it presents a less sensitive to changes in the K_N parameter into the vicinity of the fixed central point at $x_o = 1$. A direct consequence is that the pair (β_o, K_{NIP}) can be accurate determined from the data, while the parameter K_N is only poorly estimated. Another important point of view is that the sensibility for β_o increases when the offset decreases, while the K_{NIP} parameter has an opposite behavior. From this point o f view we can say that the search procedure for emergence angle should be made weighting the near-offset data, and for NIP curvature it should weight the far-offset data.

Another view of this sensibility analysis can be found through the Figure 3a,b,c.In the Figure (3), the MFS stack surface is calculated by formula (1), using the true parameters K_{NIP}, K_N and β_o, with constant velocity (Figure 1), and represented by the black surface. The other two stack surfaces are calculated using values of wavefront parameters, that correspond to plus and minus fifty percent of the exact values. In the upper part (Figure 3a) we have stack surfaces for values of β_o, in the middle part (Figure 3b) for K_{NIP} and in the bottom (Figure 3c) for K_N.

The semblance function analysis was obtained by using the set of synthetic seis-

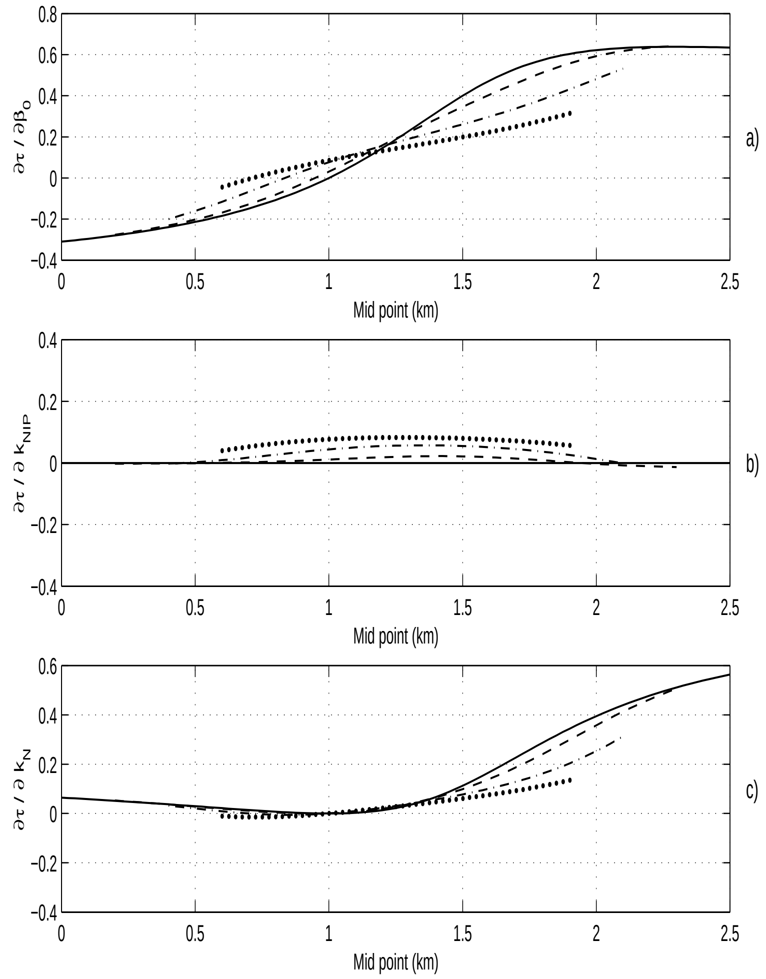

Figure 2: First derivatives of the MFS traveltime: a) With respect to β_o; b) with respect to K_{NIP}; and c) with respect to K_N. $h = 0.0$ Km (solid line), $h = 0.4$ Km (dashed line), $h = 0.8$ Km (dash-dotted line) and $h = 1.2$ Km (dotted line).

mic data generated by the software SEIS88, with an added noise corresponding to ten percent of the maximum amplitude in the data. In this experiment we have used 101 common-shots, beeing each shot separated by 25 m, 48 geophones separated by 25 m, and sample interval of 2 ms. The source pulse is a Gabor wavelet of 50 Hz dominant frequency. An example of this set of data can be seen in the Figure 4 by a commom-offset section.

In that Figures we have seen the same sensibility behavior as found early by the traveltime derivative, once again we have the K_N parameter is the worst determined. These conclusions are confirmed when we observe the semblance function behavior for each one of wavefront parameter intervals. The pair (β_o, K_{NIP}) is very good determined, while K_N is less accurate.

CONCLUSIONS

By using derivatives of the multifocusing traveltime stack formula, we present a sensibilty analysis of the functional that simulates the observed data with respect to the searched-for wavefront parameters. The most important results are the very high sensibility of the multifocusing traveltime on relation to β_o and K_{NIP}. This is an indicator that both can be very well determined by the inverse problem solution. In the other side we have seen the K_N parameter presents a strong ambiguity and is poorly determined during the optimization procedure. The non sharpness of the semblance function (Figure 3f) suggests the need to use some constraint to better determining the K_N parameter.

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APPENDIX A

For understanding the mathematical relations obtained by the multifocusing traveltime derivatives with respect to the parameters K_{NIP}, K_N and β_o, we describe each one of them as follows:

$$\frac{\partial \tau}{\partial K_N} = T_1 + T_2 \tag{A-1}$$

$$T_1 = \frac{-(1-\gamma)}{v\left(K_N - \gamma K_{NIP}\right)^2} C_1 + \frac{(1-\gamma)}{2v\left(K_N - \gamma K_{NIP}\right)}\left(C_1^{-1}\right)\left(C_3\right) + \frac{(1-\gamma)}{v\left(K_N - \gamma K_{NIP}\right)^2} \tag{A-2}$$

$$T_2 = \frac{-(1+\gamma)}{v\left(K_N + \gamma K_{NIP}\right)^2} C_2 + \frac{(1+\gamma)}{2v\left(K_N + \gamma K_{NIP}\right)}\left(C_2^{-1}\right)\left(C_4\right) + \frac{(1+\gamma)}{v\left(K_N + \gamma K_{NIP}\right)^2} \tag{A-3}$$

$$\frac{\partial \tau}{\partial K_{NIP}} = T_3 + T_4 \tag{A-4}$$

$$T_3 = \frac{(1-\gamma)}{v\left(K_N - \gamma K_{NIP}\right)^2}(\gamma)\, C_1 + \frac{(1-\gamma)}{2v\left(K_N - \gamma K_{NIP}\right)}\left(C_1^{-1}\right)\left(C_5\right) - \frac{(1-\gamma)(\gamma)}{v\left(K_N - \gamma K_{NIP}\right)^2} \tag{A-5}$$

$$T_4 = \frac{-(1+\gamma)}{v\left(K_N + \gamma K_{NIP}\right)^2}(\gamma)\, C_2 + \frac{(1+\gamma)}{2v\left(K_N + \gamma K_{NIP}\right)}\left(C_2^{-1}\right)\left(C_6\right) + \frac{(1+\gamma)(\gamma)}{v\left(K_N + \gamma K_{NIP}\right)^2} \tag{A-6}$$

$$\frac{\partial \tau}{\partial \beta_o} = T_5 + T_6 \tag{A-7}$$

$$T_5 = \frac{(1-\gamma)}{2v\left(K_N - \gamma K_{NIP}\right)}\left(C_1^{-1}\right)C_7 \tag{A-8}$$

$$T_6 = \frac{(1+\gamma)}{2v\left(K_N + \gamma K_{NIP}\right)}\left(C_2^{-1}\right)C_8 \tag{A-9}$$

The C_i parameters found in the expressions of the derivatives are given by:

$$C_1 = \sqrt{1 + \frac{2\sin\beta_o K_N(x_m-h-x_o)}{1-\gamma} - \frac{2\sin\beta_o \gamma K_{NIP}(x_m-h-x_o)}{1-\gamma} + \frac{(x_m-h-x_o)^2(K_N-\gamma K_{NIP})^2}{(1-\gamma)^2}}$$

$$C_2 = \sqrt{1 + \frac{2\sin\beta_o K_N(x_m+h-x_o)}{1+\gamma} + \frac{2\sin\beta_o \gamma K_{NIP}(x_m+h-x_o)}{1+\gamma} + \frac{(x_m+h-x_o)^2(K_N+\gamma K_{NIP})^2}{(1+\gamma)^2}}$$

$$C_3 = \frac{2\sin\beta_o(x_m-h-x_o)}{1-\gamma} + \frac{2(x_m-h-x_o)^2(K_N-\gamma K_{NIP})}{(1-\gamma)^2}$$

$$C_4 = \frac{2\sin\beta_o(x_m+h-x_o)}{1+\gamma} + \frac{2(x_m+h-x_o)^2(K_N+\gamma K_{NIP})}{(1+\gamma)^2}$$

$$C_5 = -\frac{2\sin\beta_o(\gamma)(x_m-h-x_o)}{1-\gamma} - \frac{2(x_m-h-x_o)^2(K_N-\gamma K_{NIP})(\gamma)}{(1-\gamma)^2}$$

$$C_6 = \frac{2\sin\beta_o(\gamma)(x_m+h-x_o)}{1+\gamma} + \frac{2(x_m+h-x_o)^2(K_N+\gamma K_{NIP})(\gamma)}{(1+\gamma)^2}$$

$$C_7 = \frac{2\cos\beta_o K_N(x_m-h-x_o)}{1-\gamma} - \frac{2\cos\beta_o(\gamma)K_{NIP}(x_m-h-x_o)}{1-\gamma}$$

$$C_8 = \frac{2\cos\beta_o K_N(x_m+h-x_o)}{1+\gamma} + \frac{2\cos\beta_o(\gamma)K_{NIP}(x_m+h-x_o)}{1+\gamma}$$

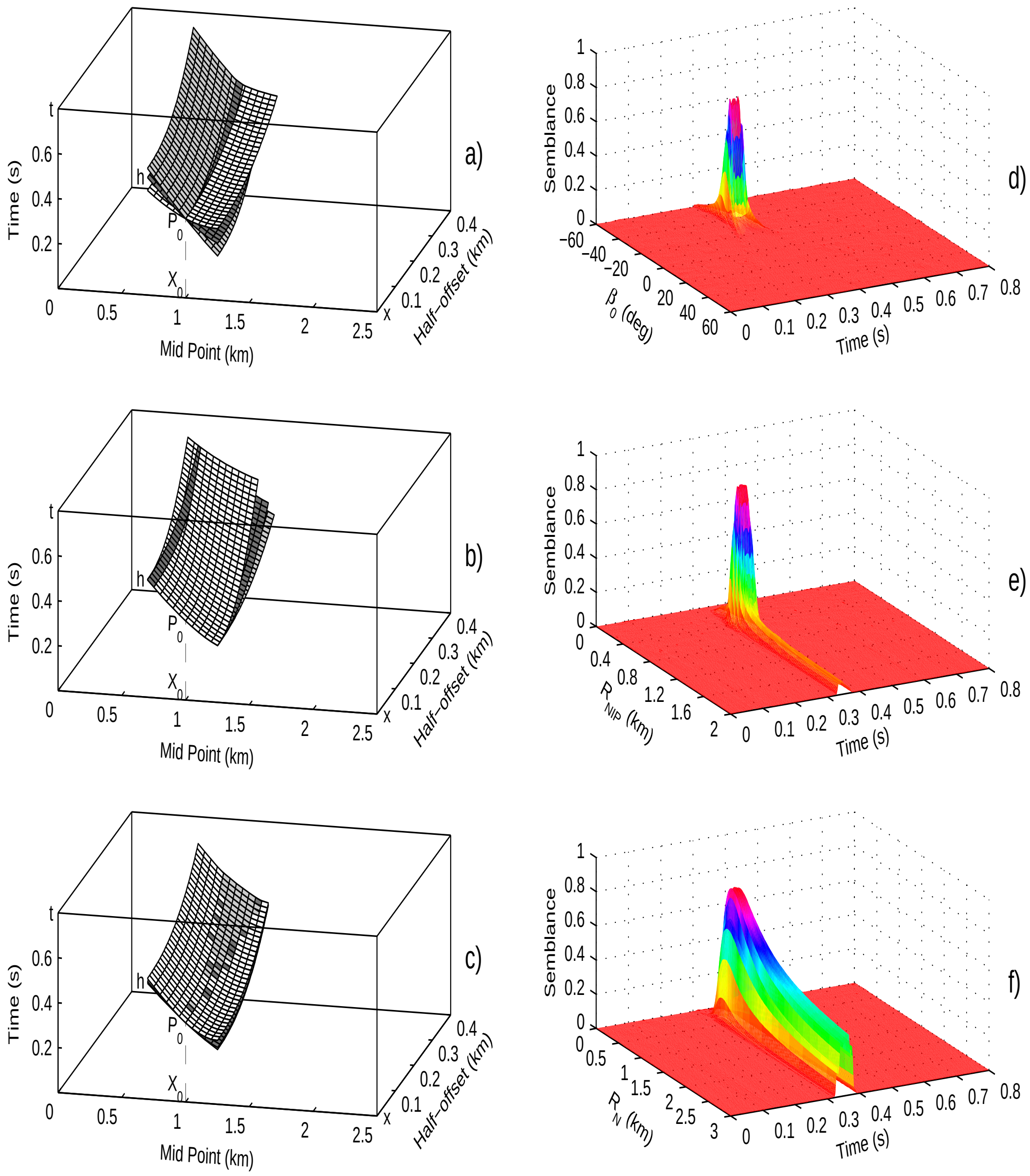

Figure 3: On the left we have the behavior of the MFS stack surface when the wavefront parameter deviates from the exact value (black surface): a) Emergence angle β_o; b) radius of curvature R_{NIP}; and c) radius of curvature R_N. On the right we have the semblance function calculated for several values of wavefront parameters: d) Emergence angle β_o; e) radius of curvatue R_{NIP}; and f) radius of curvature R_n.

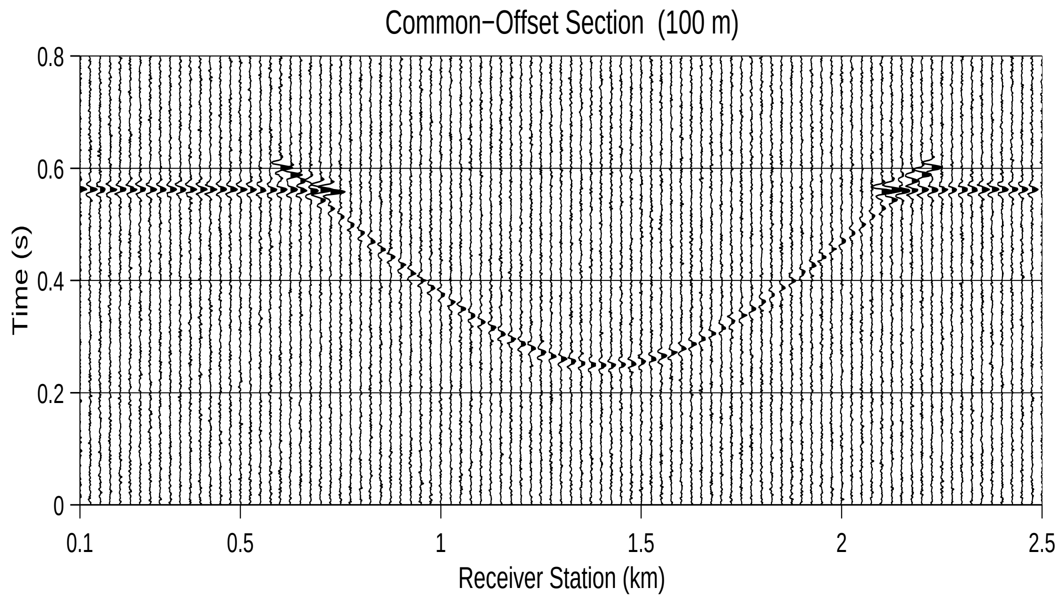

Figure 4: Example of the synthetic seismic data with noise (s/r=10) used to obtain the semblance analysis presented in the Figure 3.