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Fast computation of 3D traveltimes and migration weights using a wavefront oriented ray tracing technique

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ABSTRACT

We present a wavefront oriented ray-tracing (WORT) technique for a fast computation of traveltimes and migration weights in a smooth 3D velocity model. In this method, we propagate a wavefront stepwise through the model and interpolate output quantities (ray quantities, e.g., traveltimes, slownesses) from rays to gridpoints. In contrast to Vinje's wavefront construction method, our technique is based only on kinematic ray tracing. We show that kinematic ray tracing is sufficient for the computation of migration weights. We increase the computational efficiency by (1) using only kinematic ray tracing, (2) defining the input quantities (velocity and it's first derivatives) on a fine grid and using a linear interpolation to estimate these quantities at arbitrary points, (3) computing and interpolating only quantities which are needed for amplitude-preserving migration. For a better accuracy, in the WORT technique, we generate new rays directly at the source point (not on the wavefront). The WORT method computes multi-valued traveltimes. The maximal number of computed later-arrivals can be defined as an input parameter. By computing only first-arrival traveltimes, the WORT technique is faster than Vidale's finite-difference eikonal solver. The WORT method can be used for amplitude-preserving migration of seismic waves for data in common shot/receiver gathers.

INTRODUCTION

In this paper, we present a method for a fast computation of traveltimes and migration weights for 3D amplitude-preserving Kirchhoff prestack depth migration. Prestack depth migration is a standard method for imaging complex geology. If a subsequent AVA analysis is required, then it is necessary to use proper migration weights. Starting from results proposed by Schleicher et al. (1993) and using paraxial ray theory (Hubral

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et al., 1992), Hanitzsch (1997) expressed weight functions with respect to quantities of the Green functions. This notation is well suited for numerical implementation if the Green's functions are computed with ray tracing (Hanitzsch, 1997). In contrast to finite-difference eikonal solver (e.g., Vidale, 1990), ray tracing also allows us to compute later arrivals. Later arrivals are important for the quality of the migrated images (Geoltrain and Brac, 1993; Ettrich and Gajewski, 1996).

For 3D true-amplitude ('true-amplitude' as defined by Schleicher et al. (1993)) prestack depth migration, a huge number of two-point ray-tracing problems need to be solved. Cervený et al. (1984) described a shooting method in which a fan of rays are traced between source and receiver surface, and paraxial extrapolation is used to estimate quantities at receiver points. The main problem with this approach is the limited control of continuous illumination. Vinje et al. (1993, 1996a, 1996b) solved this problem by grouping adjacent rays into cells and by using these rays to propagate the wavefront. They used dynamic ray tracing (DRT) for the interpolation of new rays on the wavefront and for the interpolation of kinematic and dynamic quantities to gridpoints. For the representation of the model they used a set of heterogeneous layers separated by smooth interfaces. Ettrich and Gajewski (1996) showed that for the needs of prestack depths migration a gridbased model representation is more suitable and they used this in a wavefront construction (WFC) method for 2D smooth media. All the above cited methods are based on DRT.

In this paper, we propose a 3D wavefront oriented ray tracing (WORT) technique which is particularly designed for fast computation of 3D traveltimes and migration weights. We show that the weight function proposed by Hanitzsch (1997) can be approximately computed using only kinematic ray tracing (KRT). Therefore, we implement the WORT technique by using only KRT and we compute and store only the quantities which are needed for true-amplitude migration.

In the following section, we will prove that migration weights can be computed from quantities obtained by KRT, then we will present the WORT method and finally we will show a result of traveltime computation in a complex velocity model.

MIGRATION WEIGHTS

By using proper migration weights the amplitudes in migrated images determine the reflectivity. There are different theoretical approaches to true-amplitude migration / inversion (see e.g., Hanitzsch, 1997 and references therein) and the weight functions are given in different notations. Hanitzsch (1997) showed that all these expressions are related and suggested a notation in terms of the point source propagator $\mathbf{Q}_2$ (Cervený, 1985), which is well suited for numerical implementation. By considering the KMAH index, the weight function for common shot reads:

$$w = \cos\theta_G \sqrt{\rho_S \rho_G v_S v_G} \sqrt{\frac{\det \mathbf{Q}_{2,SM}}{\det \mathbf{Q}_{2,GM}}}\, e^{-i\frac{\pi}{2}(k_{SM}+k_{GM})}, \tag{1}$$

where ρ is the density, v is the velocity, θ is the inclination angle, k is the KMAH index. The subscript S denotes the source point, G denotes the receiver, M denotes the depth point under consideration (image point), SM denotes the ray segment source-image point and GM denotes the ray segment receiver-image point.

For a point source, using three near by rays we approximately write det$(\mathbf{Q}_{2,SM})$ as a function of quantities that can be computed with KRT. The expression reads:

$$\det \mathbf{Q}_{2,SM} \approx \frac{\mathbf{x}^{(21)} \times \mathbf{x}^{(31)}}{\mathbf{p}_S^{(21)} \times \mathbf{p}_S^{(31)}}, \tag{2}$$

where $\mathbf{x}^{(lm)}$ is the distance vector between point $M^{(l)}$ and $M^{(m)}$, $\mathbf{p}_S^{(lm)} = \mathbf{p}_S^{(m)} - \mathbf{p}_S^{(l)}$. l and m take values from 1 to 3 and indicate the ray under consideration, $M^{(l)}$ is the intersection of the ray with a given wavefront and $\mathbf{p}_S$ is the slowness vector at the source point (see Figure 1). The expression for det$(\mathbf{Q}_{2,GM})$ is similar.

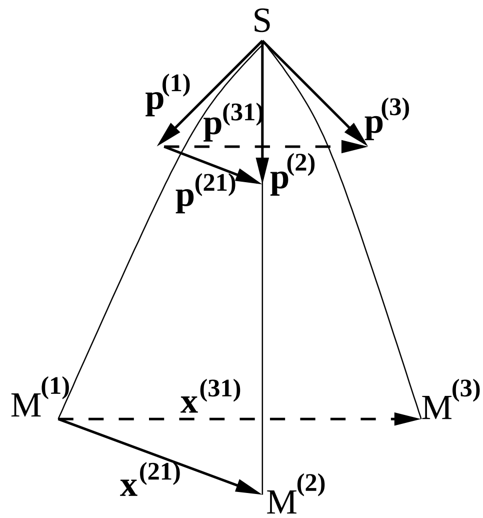

Figure 1: Distance vector $\mathbf{x}^{(lm)}$, and slowness difference $\mathbf{p}^{(lm)}$. $\mathbf{p}^{(lm)} = \mathbf{p}^{(m)} - \mathbf{p}^{(l)}$, where, l and m take values from 1 to 3 and indicate the ray under consideration. $M^{(l)}$ is the intersection of the ray with a given wavefront and $\mathbf{p}$ is the slowness vector at the source point.

WAVEFRONT ORIENTED RAY TRACING

In the WORT technique, a wavefront (which is defined by the endpoints of rays) is propagated stepwise through the model, and ray quantities are interpolated to a discrete grid. To describe the WORT technique, we compare it with the classical 3D WFC method by Vinje et al. (1996a, 1996b). In both methods the ray field is decomposed into elementary cells (a cell is the region between three adjacent rays and two consecutive wavefronts). However, the propagation of the wavefront, the generation of new rays, the estimation of ray quantities at gridpoints and the representation of the model are different.

Please note the difference between the fine grid where the input quantities are defined and the coarse grid where output quantities are estimated.

Propagation of wavefronts

The WORT technique is based on KRT, while the WFC method is based on KRT and DRT. The ray is defined by its initial conditions, e.g., for a point source the inclination and declination at the source. For a given ray, KRT is used to compute the wavefront location and the slowness vector at given traveltimes. Dynamic quantities along the same ray are computed with DRT. For more details about KRT and DRT refer to, e.g., Cervený (1985).

On the one hand, DRT is a useful tool for the interpolation of new rays on the wavefront, for the estimation of kinematic quantities at gridpoints and for the computation of migration weights. On the other hand, DRT needs a velocity model with smooth second derivatives, while for KRT smooth first derivatives are sufficient; the computational efficiency of ray tracing is reduced because the DRT system must be solved (in addition to the KRT system) along each ray, and the interpolation of new rays using DRT is not accurate in certain situations (see below) .

In section Migration weights, we have shown that migration weights can be approximately computed without DRT. We will show below that DRT is also not necessary for the generation of new rays and for the estimation of ray quantities at the migration grid.

Generation of new rays

A new ray is generated between two rays of the same cell (parent rays) if either of the following conditions is satisfied: (1) the distance between these rays is larger than a predefined maximum value, or (2) the angle between the slowness vectors of the two rays is larger than a predefined threshold.

In the WORT technique a new ray is generated directly at the source point and KRT is used to propagate the ray to the given traveltime, while in the WFC method a new ray is interpolated on a wavefront using DRT and the paraxial ray theory (e.g., Cervený, 1985). The generation of a new ray at the source point leads to more accurate ray parameter than the interpolation of a new ray on the wavefront. The accuracy of the ray parameter is essential for the accuracy of the whole method.

The interpolation of new rays on the wavefront in the WFC method is rather cumbersome. Many kinematic and dynamic quantities need to be interpolated and this affects the computational efficiency. The accuracy of the interpolation depends on the accuracy of the paraxial approximation. Accuracy of paraxial interpolation is low if the distance between two rays increase very fast or if the slowness direction change very fast. Moreover, the dynamic quantities cannot be interpolated by using the paraxial approximation and therefore a linear interpolation is used. Inexact dynamic quantities affects the interpolation of ray quantities to receiver points and the interpolation of new rays.

In the WORT technique we generate a new ray directly at the source point. The take-off slowness of the new ray is given by the average value of the two parent rays. The propagation of the ray is done by KRT with a Runge-Kutta method using an adaptive timestep. This method is fast and provides an accurate slowness and location on the wavefront for the generated ray. If rays cannot be traced in a part of the model (shadow zone) then a new ray is linearly interpolated on the wavefront. However, this is seldom required, e.g., in the complex 3D Marmousi model (see below) this option was never used.

Interpolation of ray quantities to gridpoints

In the WORT technique we interpolate the traveltime, the slowness, the take-off slowness and $\det(\mathbf{Q}_2)$ to a a discrete grid. The WORT technique computes multi-valued traveltimes. To keep migration efficient, we store only one arrival per KMAH index, and we restrict the number of arrivals by defining the maximum computed KMAH index as an input parameter. In Coman and Gajewski (2000) we demonstrated that the estimation of ray quantities at gridpoints is very time consuming. To increase computational efficiency in the WORT technique, the interpolation is done on a coarse grid. Moreover, avoiding DRT, a less number of ray quantities needs to be interpolated.

Representation of the model

In the WORT technique the velocity model and it's first derivatives are defined on a discrete fine grid. For the evaluation of velocities at arbitrary points, we use a trilinear interpolation, while in the WFC method a spline interpolation is used. The trilinear interpolation is faster than the spline interpolation and for a smooth model defined on fine grids, the difference between them is very small (Ettrich and Gajewski, 1996).

Numerical example

We test the WORT method in a 3D version of the Marmousi model. This model is an extension of a 2D smooth Marmousi model into the third dimension. The Marmousi model (Versteeg and Grau, 1990) has been widely used as a reference model to validate new methods. The velocity grid is resampled to 12.5 m in each direction and the output quantities are interpolated at every fourth gridpoint. Figure 2 shows first-arrival isochrones computed with the WORT technique. The computation of first-arrival traveltimes with the WORT technique was faster and more accurate than the computation with Vidale's (Vidale, 1990) finite difference eikonal solver. The main reason for this (unexpected) result is the fact that in Vidale's method the traveltime need to be computed on a fine grid, while in the WORT technique a more coarser grid can be chosen.

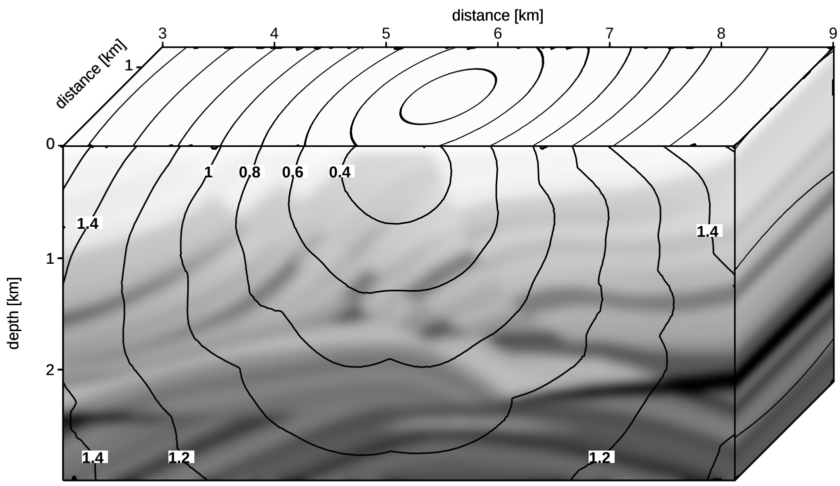

Figure 2: First arrival traveltimes in the 3D Marmousi model.

CONCLUSIONS

We have presented a method based on KRT which is particularly designed for a fast computation of 3D traveltimes and migration weights. In this method, a wavefront is propagated stepwise through the model. Output quantities (traveltimes, slownesses, det$(\mathbf{Q}_2)$) are interpolated to a coarse grid in a region between three adjacent rays and two consecutive wavefronts.

To maintain an accurate representation of the wavefront, we generate new rays directly at the source point. The following features of the WORT technique increase the computational efficiency: First, we use only KRT (avoid DRT). Second, we define the velocity model and it's first derivative on a fine grid and use trilinear interpolation to determine model parameters at arbitrary points, and third, we compute and interpolate to a coarse grid only the quantities which are needed for true amplitude migration (traveltime, slowness and $\det(\mathbf{Q}_2)$). The first-arrival traveltime computation with the WORT technique is even faster than Vidale's (Vidale, 1990) finite difference eikonal solver.

The WORT technique can be used for 3D true-amplitude migration of seismic waves for data in common shot / receiver gathers.

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