

## Common reflection surface stack: a new parameter search strategy by global optimization

G. Garabito, J. C. R. Cruz, P. Hubral and J. Costa<sup>1</sup>

**keywords:** *Imaging, zero-offset simulation, optimization*

### ABSTRACT

*By using an arbitrary source and receiver configuration a new stack processing called Common Reflection Surface was proposed in the recent years for simulating zero-offset sections. As by products, this technique provides important normal ray associated parameters, to be known: 1) The emergence angle; 2) the radius of curvature of the Normal Incidence Point Wave (NIP); and 3) the radius of curvature of the Normal Wave. It is based on the hyperbolic traveltime paraxial approximation, that is function of the three mentioned normal ray parameters. In this paper we present a new optimization strategy for determining these three parameters and simulating zero-offset section based on the CRS stack formalism. This is a three step in cascade process: 1) Two-parameters global optimization; 2) One-parameter global optimization; and 3) three-parameters local optimization. This new strategy provides high resolution simulated zero-offset sections and very good estimates of the three normal ray parameters. It is also possible to apply it, without too much additional work, this strategy for solving the conflicting dip problem.*

### INTRODUCTION

This paper concerns with the simulation of zero-offset seismic section. This is a routine seismic process based on stacking of wave field registered by multi-coverage seismic data to enhance the primary reflection events and to increase the signal-to-noise ratio.

The conventional CMP stack or NMO/DMO/STACK techniques given by (Mayne, 1962) and (Hale, 1991) use a macro-velocity model to simulate the zero-offset section. In the last years, several seismic stacking process were proposed by (Gelchinsky, 1989); (Keydar et al., 1993); (Berkovitch et al., 1994); and (Mueller, 1999), which are not dependent on the macro-velocity model, and instead of that use wavefront parameters. One of his new stack method was called Common-Reflection-Surface (CRS) by

---

<sup>1</sup>**email:** jcarlos@marajo.ufpa.br, german@ufpa.br, peter.hubral@gpi.uni-karlsruhe.de, jesse@ufpa.br

(Tygel et al., 1997). In addition to the zero-offset seismic section, the CRS method provides new stack parameters that are associated with the normal reflection ray and are useful for determination of the macro-velocity model.

The CRS stack is performed by using a hyperbolic paraxial traveltime approximation to calculate the stack surfaces as given by (Schleicher et al., 1993) and (Tygel et al., 1997). This formula is useful for seismic stacking with arbitrary source-receiver configuration in heterogeneous media. In the two-dimensional case (2-D), this approach depends on three stack parameters: (a) The emergence angle of the primary reflection normal ray ( $\beta_0$ ); (b) the radius of curvature of the normal incidence point (NIP) wave ( $R_{NIP}$ ); and (c) the radius of curvature of the normal wave ( $R_N$ ). The so-called NIP and Normal waves were defined by (Hubral, 1983), which play an important role for the second order paraxial traveltime approximation.

The CRS stack method is available by means of a coherence analysis for estimating three parameters, i.e.  $\beta_0$ ,  $R_{NIP}$  and  $R_N$ , by using an optimization strategy. (Mueller, 1999) has used a three steps optimization method. First step, one parameter search (a combination of  $\beta_0$  and  $R_{NIP}$ ) is made by using as input the multi-coverage seismic data in a Common-Mid-Point (CMP) configuration; second step, the emergence angle and the radius of curvature do the Normal wave parameters are estimated in the simulated zero-offset section obtained in the first step; and as final step, the results of the two earlier steps are used as initial approach to search the best three parameters by using the whole seismic data with arbitrary configuration. The non-linear optimization processing is based on the Simplex algorithm. (Birgin et al., 1999) modified the above optimization strategy by applying a local optimization algorithm called Spectral Projected Gradient Method.

In this paper we present a new strategy for searching the three CRS stack parameters. This is performed also in three steps: 1) Two-dimensional global optimization for determining  $\beta_0$  e  $R_{NIP}$ , simultaneously; 2) One-dimensional global optimization for determining only  $R_N$ ; and 3) Three-dimensional local optimization for the three parameters, simultaneously, using as initial approximation the results of the earlier two steps. In the first two steps the Simulated Annealing algorithm is used, and the Variable Metric algorithm is used in the last step.

## HYPERBOLIC TRAVELTIME APPROXIMATION

The second order hyperbolic expansion approaches the traveltimes of the rays in the vicinity of a fixed normal ray (called zero-offset central ray), can be derived by means of paraxial ray theory following (Schleicher et al., 1993). For two-dimensional media, and assuming the near surface velocity is known, the hyperbolic traveltime approximation can be expressed as function of three independent parameters, i.e. the emergence angle  $\beta_0$  of the normal ray, the radius of curvature  $R_{NIP}$  of the NIP wave and the

radius of curvature  $R_N$  of the Normal wave. These three parameters are measured at the emergence point of the zero-offset central ray. The NIP wave (NIP-wave) and the Normal wave are fictitious waves defined by (Hubral, 1983), which are important concepts for the seismic imaging theory. The NIP-wave is an upgoing wave that originates at the normal incidence point (NIP) of the zero-offset central ray on the reflector. The N-wave is an exploding reflector wave with an initial wavefront curvature equals to the local curvature of the reflector on the vicinity of the normal incidence point. The general expression of the hyperbolic traveltimes that can be applied to heterogeneous media with arbitrary source and receiver configuration, is expressed by

$$t^2(x_m, h) = \left( t_0 + \frac{2\sin\beta_0}{v_0}(x_m - x_0) \right)^2 + \frac{2t_0\cos^2\beta_0}{v_0} \left( \frac{(x_m - x_0)^2}{R_N} + \frac{h^2}{R_{NIP}} \right) \quad (1)$$

where  $t_0$  is the zero-offset traveltimes and  $v_0$  is the near-surface velocity;  $x_m$  and  $h$  are the coordinates of the midpoint and half-offset between the source and the receiver, respectively; and  $x_0$  is the coordinate at the emergence point of the zero-offset central ray. The  $P_o(x_0, t_0)$  is the reference point of the zero-offset section to be simulated.

The physical interpretation of the three parameters ( $\beta_0$ ,  $R_{NIP}$  and  $R_N$ ), even in heterogeneous media, is: The emergence angle of the zero-offset ray defines the angular orientation of the reflector at the NIP point on the reflector. The radius of curvature of the NIP-wave yields information about the distance between the NIP reflection point and the emergence point of the zero-offset ray. The radius of curvature of the N-wave allows information about the shape (or local curvature) of the reflector, at the NIP point. In the particular case when  $R_N = R_{NIP}$ , the reflector element collapses into a diffraction point, and no information about the shape of the scatterer is possible. Applying this condition in equation (1), it results

$$t^2(x_m, h) = \left( t_0 + \frac{2\sin\beta_0}{v_0}(x_m - x_0) \right)^2 + \frac{2t_0\cos^2\beta_0}{v_0 R_{NIP}} \left( (x_m - x_0)^2 + h^2 \right). \quad (2)$$

The hyperbolic traveltimes become a two-parameter expression ( $\beta_0$  and  $R_{NIP}$ ). For heterogeneous media, the so-called CRS stacking operator calculated by formula (2), is similar to the pre-stack Kirchhoff migration operator.

## 2-D CRS STACK

By considering a reference point of the zero-offset section  $P_o(x_0, t_0)$ , the stacking surface of the CRS stack, also called CRS stacking operator, is an approach to the kinematic response of the reflection curved interface in a heterogeneous media.

The CRS stack is a method that simulates a zero-offset section by using multi-coverage seismic data, summing the reflection events in the stacking surface, which is defined by means of the hyperbolic travelttime formula. Therefore, the definition of the best stacking surface by means of the kinematic formula (1) requires the determination of the optimal triplet parameter ( $\beta_0$ ,  $R_{NIP}$  and  $R_N$ ), for each point  $P_o(x_0, t_0)$  of the zero-offset section.

For a given point  $P_o$  of the zero-offset section, the three parameters ( $\beta_0$ ,  $R_{NIP}$  and  $R_N$ ) of the CRS stacking operator can be determined from the input data, by means of some multi-parameter search process that uses as objective function a certain coherence measure. Then, the zero-offset section is obtained by stacking the data, using the three parameters that produce the largest coherence value.

The used coherency criterion is the semblance measure given by (Neidell and Taner, 1971),

$$S = \frac{\sum_{j=k-\frac{N}{2}}^{k+\frac{N}{2}} \left( \sum_{i=1}^M f_{i,j(i)} \right)^2}{M \sum_{j=k-\frac{N}{2}}^{k+\frac{N}{2}} \sum_{i=1}^M f_{i,j(i)}^2}, \quad (3)$$

where  $f_{i,j(i)}$  is the seismic signal amplitude indexed by the channel order number,  $i = 1, \dots, M$ , and the stacking trajectory,  $j(i) = k - (N/2), \dots, k + (N/2)$ . The stacking trajectory is determined by the CRS stack formulas. The index  $M$  is the number of seismic traces and  $(N + 1)$  is the number of time gate samples.  $k$  is the index of the amplitude in the center of the time gate. The semblance function  $S$  means the ratio of signal energy to total energy of all members of the gather. It constitutes a normalized coherence measure with values between 0 and 1, and it equals to unity only if all signals in whole traces are identical.

The main problem of the CRS stack method is to find the optimal triplet parameter that maximizes the objective function. The parameter spaces are the mathematical intervals  $-\pi/2 < \beta_0 < \pi/2$  and  $-\infty < R_{NIP}(R_N) < +\infty$ . This problem is solved by a multi-dimensional global optimization algorithm, which is capable to find the global maximum, but stated at a low computational cost. However, the convergence to find the global maximum is, in general, highly dependent on the behaviour of the objective function.

If a particular zero-offset event is composed by interference of several events, we have to take into account more than one maximum. In other words, to solve the conflicting dip problem it must be used more than one triplet parameter in the stacking process. In this case, it is necessary to find the global maximum and at least a local maximum at points of the zero-offset section.

In the following, based on the formulas (1) and (2), we describe the new CRS stack

strategy by combining global and local optimization algorithms, for determining the best triplet parameter.

## **CRS STACK PROCESSING STRATEGY**

We present in this item a description of the optimization strategy that is product of our endeavour for determining the three parameters of the CRS stack operator. It is separated in three steps, being the first two performed by using a global optimization Simulated Annealing algorithm given by (Sem and Stoffa, 1995), and the third step by a local optimization Variable Metric algorithm given by (Bard, 1974). In all three steps the input data are common-offset seismic sections.

### **Optimization Strategy**

#### **Step I : Two-parameters global search**

We select a point  $P_0$  in time domain that is used as reference for determining the two parameters  $\beta_0$  and  $R_{NIP}$ , subject to  $R_{NIP} = R_N$  in the formula (1) that implies in formula (2). By using formula (3) as objective function, the inverse problem is formulated to estimate the best  $\beta_0$  and  $R_{NIP}$  with the maximum semblance value. For solving it we use a global optimization Simulated Annealing algorithm, with the initial approximation being random values into a priori defined physical intervals. We have as results in this step: 1) Maximum coherence section; 2) Emergence angle section; 3) Radius of curvature of NIP wave section; and 4) Simulated zero-offset section.

#### **Step II : One-parameter global search**

In the second step we select also a reference point  $P_0$  in time domain for determining the third parameter  $R_N$ . The values of parameters estimated by the first step are used to fix  $\beta_0$  and  $R_{NIP}$  in formula (1) in this step. In this turn, the inverse problem is formulated to estimate the best  $R_N$  with the maximum semblance value, using the objective function formula (3). We use, as before, the global optimization Simulated Annealing algorithm. In this step, the results are: 1) Maximum coherence section; 2) Radius of curvature of Normal wave section; and 3) the improved zero-offset section.

#### **Step III : Three-parameters local search**

The results of first and second steps are considered a good approach to the searched-for triplet stack parameters, and also an intermediate simulate zero-offset section. These results are to be used as initial approximation at the third step. It means that we select a reference point  $P_0$  in time domain, and using the whole data, we make a new search for the three parameters, simultaneously. The inverse problem is formulated to estimate the best triplet parameters  $(\beta_0, R_{NIP}, R_N)$  with the maximum semblance value. This problem is now solved by using a local optimization Variable Metric algorithm. As final results we have: 1) The maximum coherence section; 2)

Emergence angle section; 3) radius of curvature of the NIP wave; 3) radius of curvature of the Normal wave; and 4) the zero-offset simulated section. In Figure 1 is shown the processing flow chart for the CRS stack applied in this work.

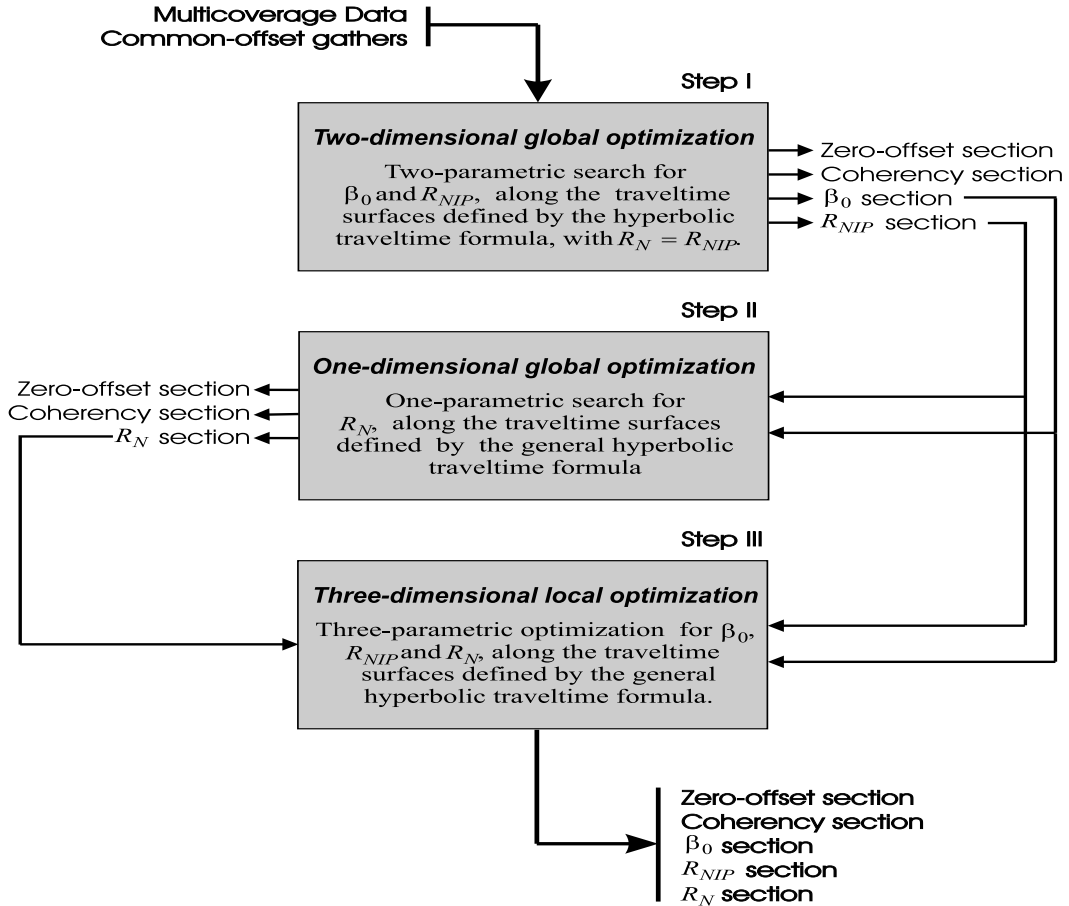


Figure 1: CRS stack processing flow chat. By the two first steps it is determined the initial triplet parameters  $(\beta_0, R_{NIP}, R_N)$ , which are used as initial guess in the final step.

## APPLICATION

By considering a synthetic model with three homogeneous layers Figure 2, we simulate multi-coverage primary reflection data using a ray tracing algorithm. The data set consists of 140 common-shots with 48 geophones separated by 25 meters. The minimum offset is 100 meters and the maximum is 1675 meters. The distance between two consecutive shots is 25 meters. The source signal is a Gabor wavelet with 40 Hz dominant frequency, and a time sample rate of 2 ms. An example of direct mod-

elled data is presented in Figure 3, that is a synthetic zero-offset seismic section with noise-to-signal ratio equals to 5.

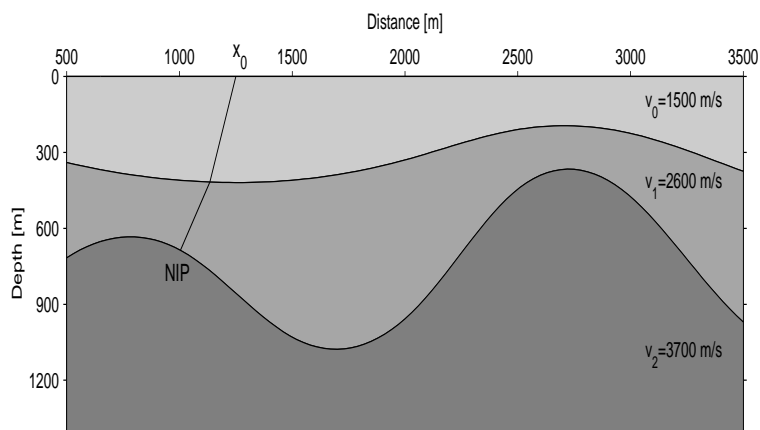


Figure 2: Synthetic model consisting of three layers.

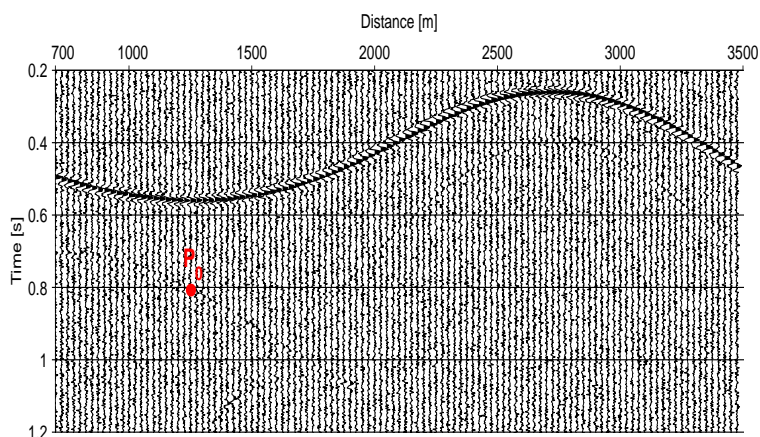


Figure 3: Modelled zero-offset section with primary reflection events corresponding to the geophysical model in Figure 2.

The common-shot sections were sorted into common-offset sections, before the optimization strategy was applied to. In Figure 4 is shown the simulated zero-offset seismic section obtained in this work by CRS stacking. Due to the CRS stack method involves a large number of traces during the stack process, the simulated zero-offset section presents a high signal-to-noise ratio, in comparison with the direct calculated zero-offset section (Figure 3).

In Figure 5 we have the maximum coherence (semblance) section corresponding to the optimized parameters at the third step. In this section, the two primary reflec-

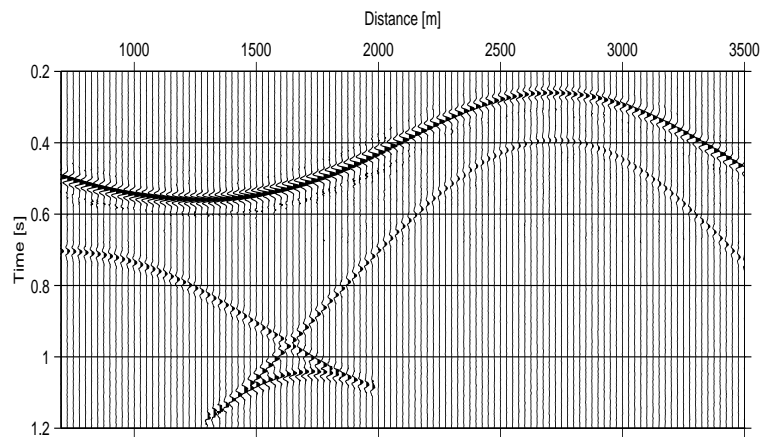


Figure 4: Simulated zero-offset section from the final CRS stack, using the three best CRS stacking parameters obtained by the third step.

tion events are identified by higher coherence values (0.3-0.9). The optimized emergence angles are presented in Figure 6, with a continuous variation in zones of reflection events. In Figure 7 we compare the CRS optimized emergence angles (cross line) and the ray theoretical calculated values (dot line).

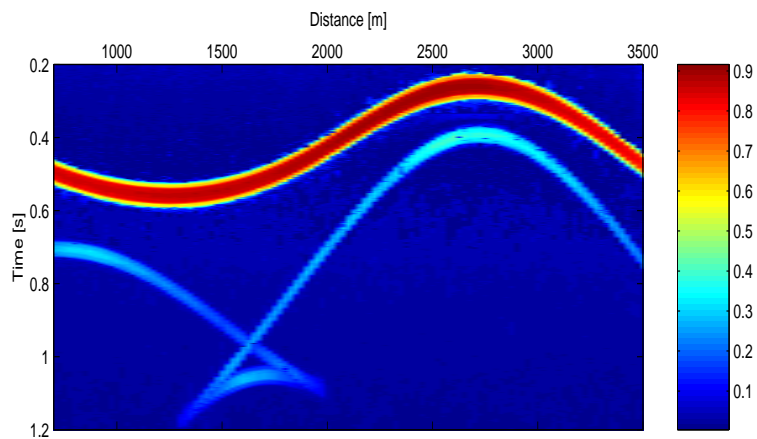


Figure 5: Coherency (semblance) section generated in the third step by the stacking surfaces using multi-coverage data.



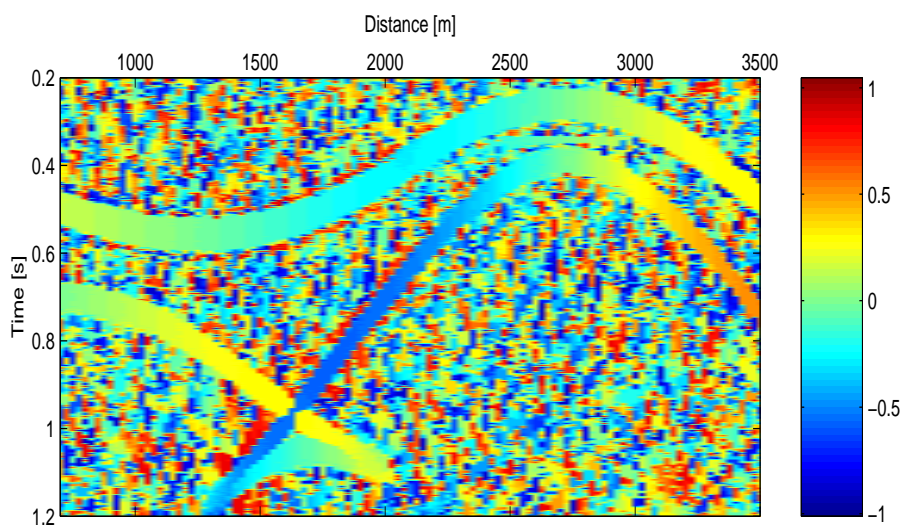


Figure 6:  $\beta_0$  section with the best estimated results of the CRS stack.

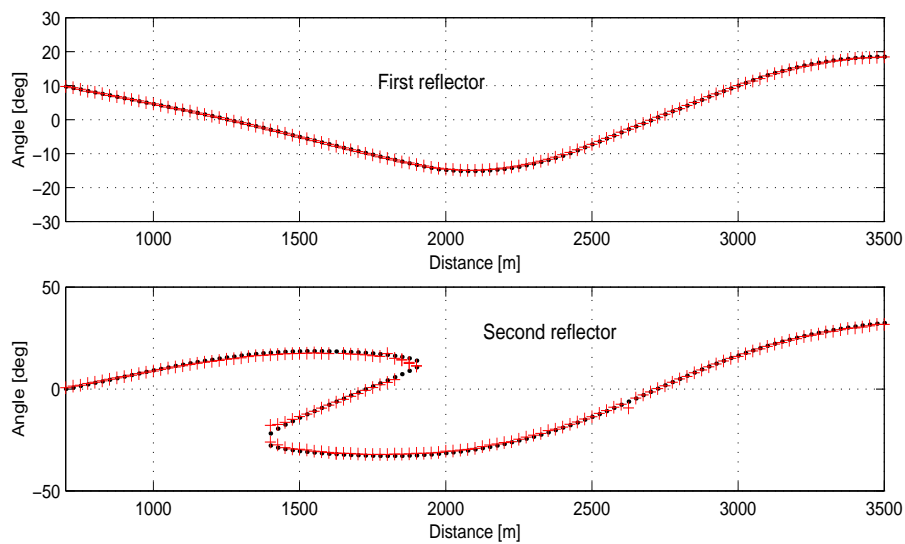


Figure 7: Comparison of  $\beta_0$ . Direct calculated values (dot line) and final estimated values (cross line). Top: First interface and bottom: second interface.

The optimized radii of curvatures of the NIP waves are shown in Figure 8. For comparison, in Figure 9, we have the optimized NIP wave radii of curvatures (cross line) and the ray theoretical calculated values (dot line). We observe that in caustic region the radius of curvature of the NIP wave is only poorly estimated. This last result is due to the destructive interference of caustic events.

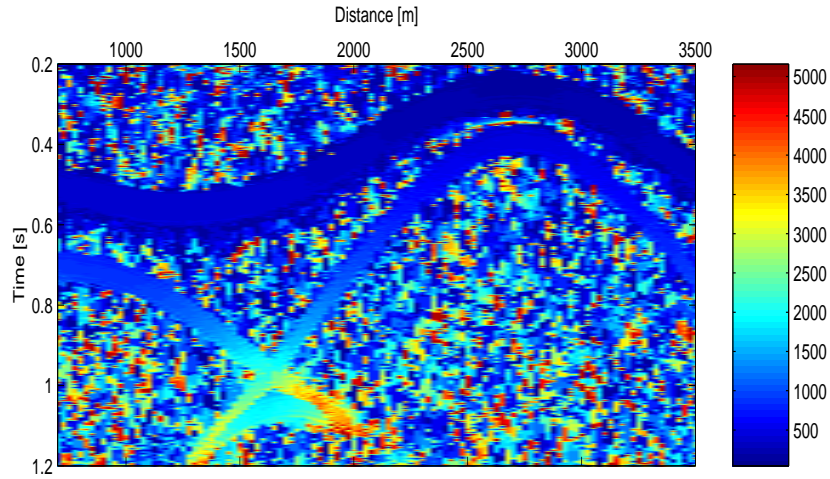


Figure 8:  $R_{NIP}$  section with the best estimated results of the CRS stack.

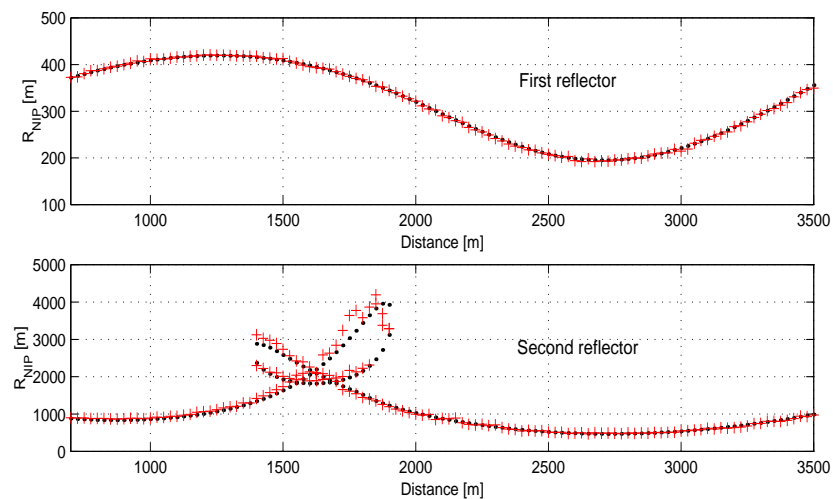


Figure 9: Comparison of  $R_{NIP}$ . Direct calculated values (dot line) and estimated values (cross line). Top: First interface and bottom: second interface.

In Figure 10 we have the optimized radii of curvatures of the Normal waves. Due to the weaker sensibility of  $R_N$  in the optimization CRS stack, we have that zones of sign changes of curvature in the synthetic model correspond to wrong optimized values in the result radius section. In Figure 11, we compare the optimized values of radius of curvature of the Normal wave (cross line), and ray theoretical calculated values (dot line), both at the reflector interfaces. This third parameter is also not good estimated in the caustic zone of the second reflector, due to the destructive interference of the caustic events.

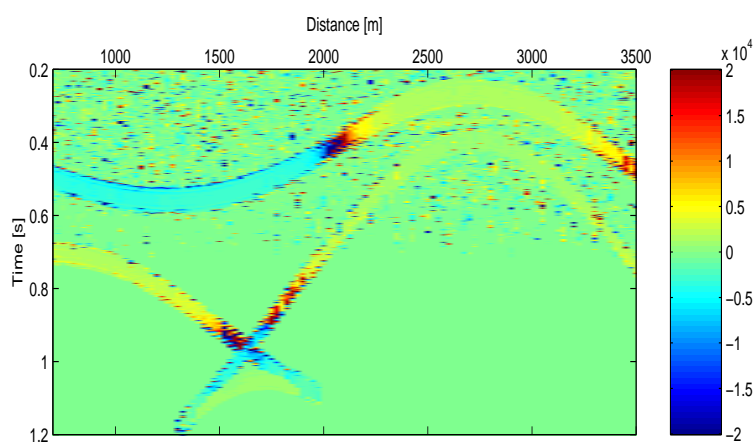


Figure 10:  $R_N$  section with the best estimated results of the CRS stack.

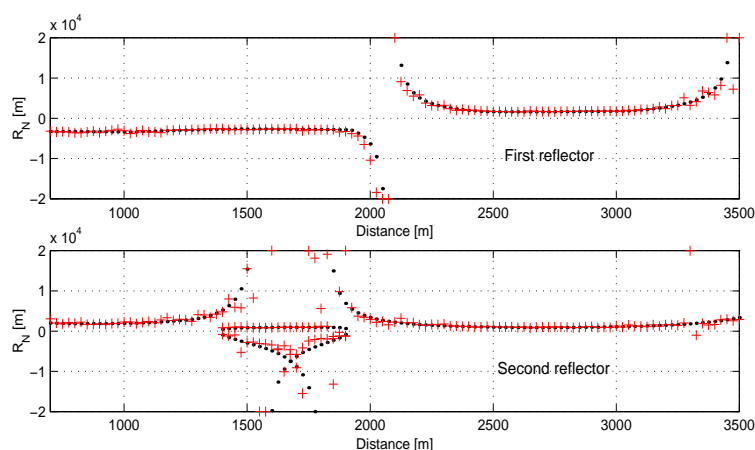


Figure 11: Comparison of  $R_N$ . Direct calculated values (dot line) and estimated values (cross line). Top: First interface and bottom: second interface.

## CONCLUSION

In this work, with a reasonable computational cost, we have shown a new strategy for determining the three parameters of the hyperbolic approximation CRS formula, by combining three steps: The first, by using a two-parameters global optimization for  $\beta_0$  and  $R_{NIP}$ , subject to  $R_N = R_{NIP}$ ; the second, a one-parameter global optimization for  $R_N$ ; and the third, the three-parameters local optimization for  $\beta_0$ ,  $R_{NIP}$  and  $R_N$ . This three steps are applied in cascade, such that we can observe a progressive improving of the parameter estimates. To apply the global optimization we have used a Simulate Annealing algorithm, and for doing the local optimization in the final step we used a Variable Metric algorithm.

This new CRS stack strategy provide us very good results when applied to multi-coverage synthetic seismic data. In the simulated zero-offset section we have a high signal-to-noise ratio, and very good resolution of primary reflection events. The emergence angle is better estimated in comparison with the other two searched-for parameters. The radii of curvatures of the NIP and Normal waves estimates have lower accuracies only in the caustic zone at the second reflector.

Finally, we can affirm this new CRS stack strategy is available to solve the conflicting dip problem, so that we have the possibility to determine the global maximum and at least one local maximum.

## ACKNOWLEDGEMENTS

We thank the Brazilian National Petroleum Agency for supporting the first author of this research. We also thank the seismic groups of Karlsruhe University and Campinas University for the fruitful discussions during the preparation of this paper, and the seismic work group of the Charles University, Prague, Tchech Republic, for making the ray tracing software SEIS88 available to us.

## REFERENCES

- Bard, Y., 1974, Nonlinear parameter estimation: Academic Press, Inc., New York,USA.
- Berkovitch, A., Gelchinsky, B., and Keydar, D., 1994, Basic formula for multifocusing stack Basic formula for multifocusing stack, Extended Abstract of the 56th EAGE.
- Birgin, G., Bilot, R., Tygel, M., and Santos, L., 1999, Restricted optimization: A clue to a fast and accuracy implementation of the common reflection surface stack method: Journal of Applied Geophysics.

- Gelchinsky, B., 1989, Homeomorphic imaging in processing and interpretation of seismic data-fundamentals and schemes Homeomorphic imaging in processing and interpretation of seismic data-fundamentals and schemes, Extended Abstract of the 59th Society Exploration Geophysics Meeting.
- Hale, D., 1991, Dip moveout processing:, volume 5 SEG.
- Hubral, P., 1983, Computing true amplitude reflections in laterally inhomogeneous earth: *Geophysics*, **48**, 1051–1062.
- Keydar, S., Gelchinsky, B., and Berkovitch, A., 1993, The combined homeomorphic stacking The combined homeomorphic stacking, Extended Abstract of the 63th Society Exploration Geophysics Meeting.
- Mayne, W., 1962, Common reflection point horizontal data stacking techniques: *Geophysics*, **27**, no. 6, 927–938.
- Mueller, T., 1999, The common reflection surface stack method - seismic imaging without explicit knowledge of the velocity model: Ph.D. thesis, Karlsruhe University, Germany.
- Neidell, N., and Taner, M., 1971, Semblance and other coherency measures for multi-channel data: *Geophysics*, **36**, 482–497.
- Schleicher, J., Tygel, M., and Hubral, P., 1993, Parabolic and hyperbolic paraxial two-point traveltimes in 3d media: *Geophysical Prospecting*, **41**, no. 4, 495–514.
- Sem, M., and Stoffa, P., 1995, Global optimization horizontal methods in geophysical inversion: Elsevier Science, Netherlands.
- Tygel, M., Mueller, T., Hubral, P., and Schleicher, J., 1997, Eigenwave based multiparameter traveltimes expansions: Eigenwave based multiparameter traveltimes expansions:, Expanded Abstract of the 67th Ann. Int. SEG.