Wave Inversion Technology, Report No. 4, pages 159-173

A weak fluctuation approximation of the primaries in typical realizations of wavefields in random media

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keywords: *Seismic waves, scattering, random media, self-averaging*

ABSTRACT

We construct the complex, ensemble averaged wavenumber of an initially plane wave propagating in 2-D and 3-D weakly heterogeneous random media. The validity range of this complex wavenumber has practically no restrictions in the frequency domain. Its real part is related to the phase velocity dispersion, whereas its imaginary part denotes scattering attenuation. These logarithmic wavefield attributes, obtained by combination of the Rytov approximation and the causality principle, are self-averaged quantities and allow to describe any typical, single realization of the wavefield. Typical wavefield realizations or seismograms are those that are nearly identical to the most probable realization. We verify the analytical results with help of finite-difference simulations in 2-D elastic random media. A statistical analysis of the simulated wavefield shows how typical realizations of seismograms can be identified. The method agrees well with numerical results.

INTRODUCTION

In seismics and rock physics the measured seismograms usually consist of the so-called primary wavefield followed by the coda. The primary wavefield is the wavefield in the vicinity of the first arrivals. Performing an average over all possible realizations of disorder one obtains the so-called coherent wave (for a recent review see Tourin et al., 2000). The coda is the long complex tail of the seismograms and contains multiple scattering contributions (Sato and Fehler, 1998).

Within the weak wavefield fluctuation regime there exist several approaches to quantify scattering attenuation. They include the meanfield theory based on the Born approximation yielding an estimator of the coherent wavefield (Keller, 1964). That the meanfield theory is not adequate to describe scattering attenuation is well-known (Wu,

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1982). There is also the so-called traveltime-corrected meanfield formalism which is successfully applied to interpret attenuation measurements in seismology (Sato and Fehler, 1998). The traveltime-corrected meanfield formalism however is based on heuristic assumptions. Stronger wavefield fluctuations visible through a longer and more complex coda indicate that the process of multiple scattering becomes important. Then the radiative transfer theory and the diffusion approximation are adequate concepts to describe averaged wavefields . In addition numerous numerical studies characterized the scattering attenuation in heterogenous structures (e.g. Frankel and Clayton, 1986).

Several pulse propagation theories for random media already appeared in the literature. Wenzel (1975) describes the propagation of coherent transients with perturbation theory. Beltzer (1989) investigated the pulse propagation in heterogeneous media based on the causality principle. The above mentioned wavefield approximations describe the wavefield for an ensemble of medium's realizations. This is because ensemble averaged wavefield attributes are used. However, in geophysics only one realization of the heterogenous medium exists. Then the ensemble averaged wavefield attribute does only represent the measured one provided that it is a self-averaged wavefield attribute (Gredeskul and Freilikher, 1990). That is to say it becomes averaged while propagating inside the random medium.

A pulse propagation theory valid for single realizations of 2-D and 3-D random media is not available. In the case of 1-D random media such a wavefield description is known as the generalized O'Doherty-Anstey (ODA) formalism (Shapiro and Hubral, 1999). It is a suitable wavefield description of the primary wavefield, i.e., the wavefield in the vicinity of the first arrival. There are also recent efforts to generalize the ODA theory for so-called locally layered media, i.e., media with layered microstructure but a 3-D heterogeneous macroscopic background (Solna and Papanicolaou, 2000).

The purpose of this study is to present a method that allows to describe pulse propagation in single realizations of 2-D and 3-D random media. It is based on ensemble-averaged, logarithmic wavefield attributes derived in the Rytov approximation which is only valid in the weak wavefield fluctuation regime. Müller and Shapiro (2000) numerically showed the partial self-averaging of logarithmic wavefield attributes and constructed the medium's Green function for an incident plane wave. Here, we follow the same strategy, however using logarithmic wavefield attributes that additionally fulfill the Kramers-Kronig relations and are valid in a very broad frequency range.

THEORY

Rytov transformation and the generalized O'Doherty-Anstey theory

The basic mathematical model for scalar wave propagation in heterogeneous media is the wave equation with random coefficients. In the following, we look for a solution of the acoustic wave equation

$$\triangle u(t,\mathbf{r}) - p^2(\mathbf{r})\frac{\partial^2 u(t,\mathbf{r})}{\partial t^2} = 0 \tag{1}$$

with $u(\mathbf{r},t)$ as a scalar wavefield (in the following simply denoted by u). Here we defined the squared slowness as

$$p^2(\mathbf{r}) = \frac{1}{c_0^2}(1+2n(\mathbf{r})) \quad , \tag{2}$$

where c_0 denotes the propagation velocity in a homogeneous reference medium. The function $n(\mathbf{r})$ is a realization of a stationary statistically isotropic random field with zero average, i.e., $\langle n(\mathbf{r})\rangle = 0$ and is characterized by a spatial correlation function $B(r) = \langle n(\mathbf{r}_1)n(\mathbf{r}_2)\rangle$ that only depends on the difference coordinate $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Later we make use the exponential correlation function $B(r) = \sigma_n^2 \exp(-|r|/a)$, where σ_n^2, a denote the variance of the fluctuations and correlation length, respectively.

We consider now a time-harmonic plane wave propagating in a 2-D and 3-D random medium and describe the wavefield inside the random medium with help of the Rytov transformation, using the complex exponent $\Psi = \chi + i\phi$, where the real part χ represents the fluctuations of the logarithm of the amplitude and the imaginary part ϕ represents phase fluctuations. Omitting the time dependence $\exp(-i\omega t)$, we write

$$u(\omega,\mathbf{r}) \;=\; u_0(\omega,\mathbf{r})\exp(\chi(\omega,\mathbf{r}) + i\phi(\omega,\mathbf{r}))\,, \tag{3}$$

where $u_0 = A_0\exp(i\phi_0)$ is the wavefield in the homogeneous reference medium ($n(\mathbf{r}) = 0$) with the amplitude A_0 and the unwrapped phase ϕ_0. To be more specific, we assume that the initially plane wave propagates vertically along the z-axis. Equation (7) can then be written

$$u(\omega, z=L, \mathbf{r}_\perp) = e^{iK(\omega,L,\mathbf{r}_\perp)L}\,, \tag{4}$$

where we introduced the complex wavenumber

$$K(\omega,L,\mathbf{r}_\perp) \;=\; \left(\frac{\phi(\omega,L,\mathbf{r}_\perp)}{L} + \frac{\phi_0(\omega,L)}{L}\right) - i\frac{\chi(\omega,L,\mathbf{r}_\perp)}{L} \tag{5}$$

$$\;=\; \varphi(\omega,L,\mathbf{r}_\perp) + i\alpha(\omega,L,\mathbf{r}_\perp)\,, \tag{6}$$

with φ and α denoting the phase increment and attenuation coefficient, respectively. L is the travel-distance and $\mathbf{r}_\perp$ denote the transverse coordinates relative to the z-axis.

For 1-D random media, Shapiro & Hubral (1999) approximated the transmitted wavefield with help of a second-order Rytov approximation. An essential property of their wavefield description is its property to describe the wavefield in a single realization of the random medium. That is to say, it is an approximation of non-averaged wavefields. This is possible due to the use of self-averaged wavefield attributes. The latter are quantities that assume their ensemble averaged values when propagating in a single realization of the random medium. Shapiro & Hubral indeed showed the self-averaging of the phase increment and the attenuation coefficient. This strategy leads to the generalized O'Doherty-Anstey theory. Now we adopt this strategy for the case of 2-D and 3-D random media. By analogy with the generalized ODA formalism in the 1-D case, we use the wavefield approximation in 2-D and 3-D:

$$u(\omega, L, \mathbf{r}_\perp) \approx e^{(\langle\varphi\rangle + i\langle\alpha\rangle)L} \quad . \tag{7}$$

Note that equation (7) is expected to describe the (primary) wavefield in single realizations of random media, since its lefthand side is not subjected to statistical averaging. The righthand side contains however the ensemble averaged wavefield attributes $\langle\varphi\rangle$ and $\langle\alpha\rangle$. To keep equation (7) physically true, we require self-averaging of the quantities α and φ. In the following we consider the approximations and properties of the logarithmic wavefield attributes $\langle\chi\rangle$ and $\langle\phi\rangle$. The self-averaging of α and φ we show in section (2.3).

Approximations for the logarithmic wavefield attributes

We derive now tractable approximations for the wavefield attributes $\langle\chi\rangle$ and $\langle\phi\rangle$ in the weak wavefield fluctuation regime. To do so we note that the wavefield can be separated into a coherent and fluctuating (incoherent) part: $u = \langle u\rangle + u_f$ where $\langle u\rangle$ denotes the ensemble averaged wavefield. A measure of the wavefield fluctuations is the ratio

$$\varepsilon = \left|\frac{u_f}{\langle u\rangle}\right| \quad . \tag{8}$$

This gives $\langle\varepsilon^2\rangle = \frac{I_t}{I_c} - 1$, where $I_t = \langle|u|^2\rangle$ is the total intensity. I_t is a constant (which is set to unity) if we ignore backscattering. $I_c = |\langle u\rangle|^2$ is the coherent intensity. The range of weak fluctuation is defined by $\langle\varepsilon^2\rangle \ll 1$ and means that the coherent intensity is of the order of the total intensity. As shown in Shapiro et al. (1996), these assumptions lead to

$$\langle\chi\rangle = -\sigma^2_{\chi\chi} + \mathcal{O}(\varepsilon^3) \quad , \tag{9}$$

$$\langle\phi\rangle = \phi_c - \phi_0 - \sigma^2_{\chi\phi} + \mathcal{O}(\varepsilon^3) \quad . \tag{10}$$

These are useful relations since all quantities on the righthand side can be obtained in the Rytov- and Bourret approximation. Explicit expressions for $\sigma^2_{\chi\chi}, \sigma^2_{\chi\phi}, \phi_c$ in 2-D and 3-D media can be found in Shapiro et al. (1996) and Shapiro & Kneib (1993).

It is the purpose of this section to construct logarithmic wavefield attributes which satisfy the Kramers-Kronig relationship. That means the primary wavefield in random media obeys the causality principle. For a comprehensive review of this topic we refer to Beltzer (1989). The Kramers-Kronig equations allow to reconstruct the attenuation from the dispersion behavior and vice versa since both quantities are related by a pair of Hilbert transforms. To this end it is sufficient to derive a Kramers-Kronig relationship for the complex wavenumber $K(\omega)$ of the plane wave response. For our purpose it is expedient to use the formulation of the Kramers-Kronig relationship of Weaver and Pao (1981):

$$\varphi(\omega') = B\omega' + \frac{2\omega'}{\pi}\int_0^\infty \frac{\alpha(\omega)-\alpha(\omega')}{\omega^2-\omega'^2}\,d\omega + \varphi(0) \tag{11}$$

$$\alpha(\omega') = \frac{-2\omega'^2}{\pi}\int_0^\infty \left[\frac{\varphi(\omega)}{\omega} - \frac{\varphi(\omega')}{\omega'}\right]\frac{d\omega}{\omega^2-\omega'^2} + \alpha(0)\,, \tag{12}$$

where $B = \lim_{\omega\to\infty}\alpha(\omega)/\omega$.
In the following we derive the scattering attenuation coefficient α by applying equation (12) to the phase increment. Let's begin with the Rytov part of the phase increment which is in 3-D random media given by equation (10) in Müller & Shapiro (2000). Inserting that into equation (12) we get after integration

$$\alpha_R(\omega') = \frac{\sigma^2_{\chi\chi}}{L}\,. \tag{13}$$

Thus the Rytov parts of equations (9) and (10) are related by the Kramers-Kronig relationship. Now the questions arises, what does happen with the Bourret part of equation (10), namely ϕ_c, when subjected to the Kramers-Kronig relation? We show that this results in an additional term α_B. To do so, we note that the phase increment resulting from the Bourret approximation can be written

$$\varphi_c(\omega') = \frac{\omega'}{c_0} + \frac{\omega'^2}{c_0^2}\int_0^\infty dr\,\sin(2\omega'/c_0 r)B(r)\,. \tag{14}$$

Inserting equation (14) into equation (12) and performing the integrations we obtain

$$\alpha_B(\omega') = 2\frac{\omega'^2}{c_0^2}\int_0^\infty B(r)\sin^2(\omega'/cr)dr - \frac{\omega'^2}{c_0^2}\int_0^\infty B(r)dr\,. \tag{15}$$

Using the relation between correlation function and fluctuation spectrum (see e.g. Rytov et al., 1987), equation (15) becomes

$$\alpha_B(\omega) = -2\pi^2\frac{\omega^2}{c_0^2}\int_{2\omega/c_0}^\infty d\kappa\Phi(\kappa)\kappa\,. \tag{16}$$

To summarize, we derived two pairs of wavefield attributes, $\{\sigma^2_{\chi\chi}, -\sigma^2_{\chi\phi}\}$ and $\{\alpha_B L, \phi_c - \phi_0\}$, each of them related by (12). Therefore, the causality principle suggests to use the following logarithmic wavefield attributes in 2-D as well as 3-D random media.

$$\langle\chi\rangle = -(\alpha_R + \alpha_B)L = -\sigma^2_{\chi\chi} - \alpha_B L \quad , \tag{17}$$

$$\langle\phi\rangle = \phi_c - \phi_0 - \sigma^2_{\chi\phi} \quad . \tag{18}$$

Note that we used in equations (13)-(16) the 3-D wavefield attributes and that equations (17) and (18) are also valid for 2-D random media if we skip κ in the integral over $d\kappa$ and divide by π.

Self-averaging of the logarithmic wavefield attributes

A self-averaged quantity tends to its mathematical expectation value provided that the wave has covered a sufficient large distance inside the medium. For 2-D and 3-D random media, we show that the attenuation coefficient α and the phase increment φ in the above discussed approximations tend to their expectation values for increasing travel-distances. In analogy to the 1-D case, we compute the relative standard deviations of the attenuation coefficient and phase increment. Using synthetic wavefield registrations of finite-difference simulations (see section 3.1) we compute the relative standard deviation of the attenuation coefficient for several travel distances. Fig. (1) displays the numerically determined $\sigma_\alpha/\alpha(\omega_0)$ as a function of L for 6 simulations (ω_0 is the center-frequency of the input wavelet, see also section 3.1). Instead of considering the relative phase increment fluctuations, we compute the relative traveltime fluctuations. This is an estimate of σ_φ/φ since $t = \varphi/\omega_0$. Again we observe the decrease of the relative standard deviation with increasing travel-distance (Fig. 2).

It is clear that within the validity range of the Rytov approximation (that implies for finite travel-distances) only a partial self-averaging can be reached. That is to say the probability densities of the wavefield attributes have not yet converged to a δ-function what would indicate that the quantities are not any more random. Therefore, it is expedient to look for the probability density of the logarithmic wavefield attributes and to introduce the concept of typical realizations of a stochastic process. Typical realizations are defined to be close to the most probable realization which is defined by the maximum of the probability density function (Lifshits et al., 1988, and Gredeskul and Freilikher, 1990). Now, the partial self-averaging means that we deal with wavefield realizations that are most likely to occur and therefore are typical realizations.

It can be theoretically and experimentally shown that χ and ϕ are normally distributed random variables (see chapter (2.5) of Rytov et al., 1987). If χ is described by the probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\eta)^2/2\sigma^2}$ (where η and σ are the mean and standard deviation, respectively), then we might expect that the Rytov transformation yields log-normal distributed wavefield amplitudes. Indeed, in section (4.2) we will numerically confirm the log-normal probability density of the wavefield amplitudes: $p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{-(\ln x-\eta)^2/2\sigma^2}$. With the above criterion, it is possible to find the typical realizations from numerical experiments by constructing the probability density functions and to compare them with the discussed wavefield attributes. The identification of typical seismograms is explained in section (3.2).

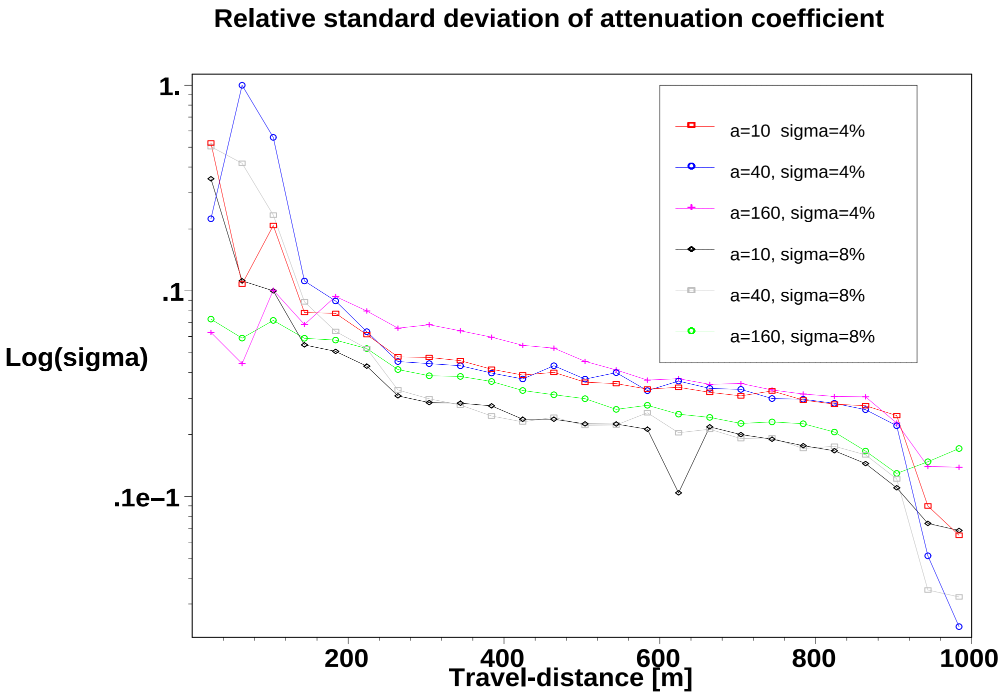

Figure 1: The relative standard deviations of the attenuation coefficient for a 2-D exponentially correlated random medium evaluated. For all investigated ratios of λ/a and standard deviations we can observe a decrease of the relative standard deviations with increasing travel-distances.

Acoustic and elastic Green's function for random media

Now, we show how finite-bandwidth pulses evolve in time when propagating in random media. In order to construct the Green's function we have to combine the results for the ensemble-averaged log-amplitude and phase fluctuations obtained in equations (17) and (18). Finally, by integration over the whole range of frequencies we obtain the time-dependent transmission response due to the initial plane wave:

$$G(t, L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, e^{-\alpha L + i\varphi L} e^{-i\omega t} . \tag{19}$$

That is what we call the Green's function for random media. It is now possible to describe seismic pulses as

$$u(t,L) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, U_0(\omega) e^{i(KL-\omega t)}\,, \tag{20}$$

where $U_0(\omega)$ is the Fourier transform of the input signal $u_0(t)$.

In order to obtain explicit analytical results for the Green's function in the time domain (that is to evaluate the integral in equation (19)), we have to introduce some simplifications. To do so, we study the behavior of equations (17) and (18) in the Fraunhofer approximation. The latter is characterized by a large wave parameter $D \gg 1$ so that the mean of phase fluctuations can be neglected. The Fraunhofer approximation becomes valid for large travel-distance L. From the behavior of scattering attenuation, it turns out that for large travel-distances only the low-frequency components of the transmitted pulse survive. Thus, with increasing L not only L/a but also $(ka)^{-1}$ effectively increases. Therefore, equation (19) can be reduced to:

$$G_{apr}(t,L) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-L\alpha_{low}} e^{i(kL-\omega t)} \quad , \tag{21}$$

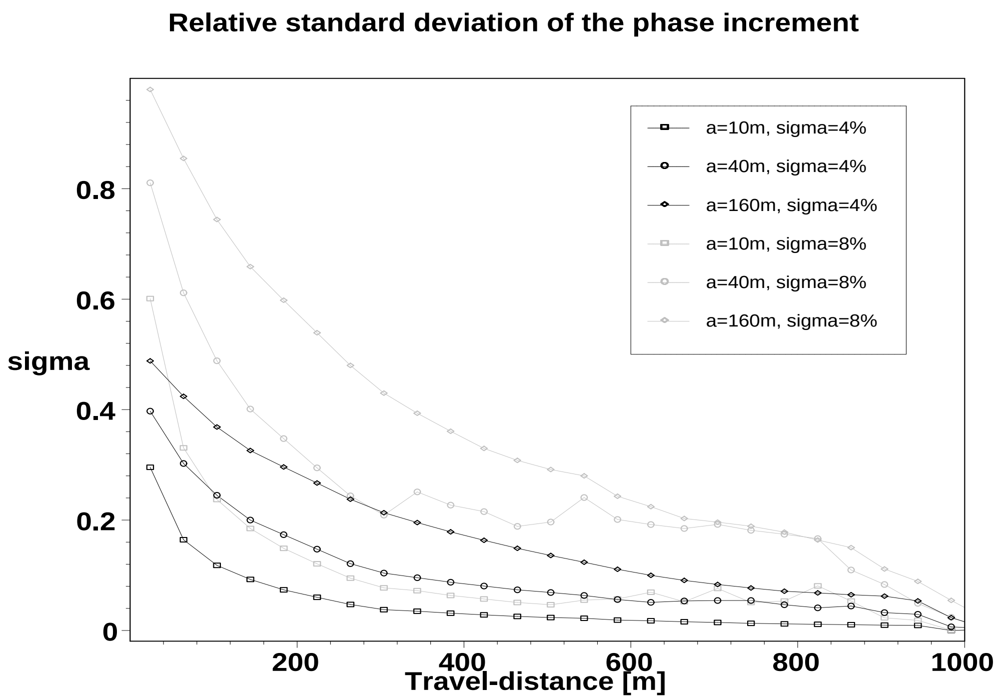

Figure 2: The relative standard deviations of the phase increment for a 2-D exponentially correlated random medium.

where α_{low} is given by

$$\alpha_{low} = 2\pi k^2 \int_0^{2k} d\kappa \, \Phi(\kappa) \quad . \tag{22}$$

Evaluating then the integral in (21) yields a Gaussian pulse (in 2-D as well as in 3-D).

$$G_{apr}(t, L) = \sqrt{\frac{A}{L}} e^{-\frac{\pi A}{L}(t - L/c_0)^2} \tag{23}$$

with $A = c_0^2/(4\pi\hat{\alpha})$ and $\hat{\alpha} = 2\pi^2 \int_0^{4L/a^2} d\kappa \, \kappa\Phi(\kappa)$. It is interesting to note that the $1/\sqrt{L}$ dependency of the pulse in 2-D and 3-D is also obtained for 1-D random media (compare with equation (7.8) in Shapiro and Hubral, 1999).

A wavefield description in elastic media should be based on the elastodynamic wave equation. Gold (1997) discussed the generalizations of the Rytov as well as the Bourret approximations to elastic media. Under the assumption of weak wavefield fluctuations, the propagation of elastic waves shows the same behavior as acoustic waves. The Rytov approximation for elastic P and S-waves yields for the complex exponent Ψ in 2-D and 3-D random media the following equation:

$$\Psi_{P,S}^{2D,3D} = 2 \begin{pmatrix} \alpha^2 \\ \beta^2 \end{pmatrix} \int n_v(\mathbf{r}') \frac{u_0(\mathbf{r}')}{u_0(\mathbf{r})} \, G^{2D,3D}(\mathbf{r} - \mathbf{r}') \, d\mathbf{r}' \quad , \tag{24}$$

where $G^{2D,3D}$ is the acoustic Green's function in 2-D/3-D and α, β are the P- and S-wavenumbers, respectively. Gold et al. (2000) applied the Bourret approximation in the Dyson equation in order to obtain the coherent Green's function in isotropic elastic random media. From this consideration the coherent phase ϕ_c is obtained and can be used for equation (10).

NUMERICAL EXPERIMENTS

FD-modeling in elastic random media

Now let us compare the analytical Green's function with finite-difference simulation results for wave propagation in 2-D elastic (isotropic) random media with an exponential correlation function of the velocity fluctuations. For the numerical computation of the wavefield we use the so-called rotated-staggered grid finite-difference scheme for the elastodynamic wave equation (Saenger et al., 2000).

We simulate a plane wave propagating from the top down to a certain depth (z-direction) in a single random medium realization. The background medium is characterized by a P-wave velocity of $v_P = 3000$ m/s, a S-wave velocity of $v_S = 1850$ m/s and a density of $\rho = 2.5\ \mathrm{g/cm^3}$. In the actual FD-model the random medium is embedded in a constant background medium with the properties defined above. For the modeling we need instead of velocities the stiffness tensor components $c_{11} = v_p^2\rho$ and $c_{55} = v_S^2\rho$ and density. For simplicity, only the stiffness tensor component c_{11} exhibits exponentially correlated fluctuations. We simulate a line-source exhibiting only a z-component. The wavelet is the second derivative of a Ricker-wavelet with a dominant frequency of about 75 Hz (this corresponds to a wavelength of 40 m for the P-wave). The wavefield is recorded by several receiver lines (where each line consists of 146 receivers) which are placed at several depths positions such that the mean propagation direction of the plane wave and the receiver lines are perpendicular.

Pulse propagation in random media

The here presented theory of pulse propagation depends very much on the ratio of wavelength and correlation length λ/a. Therefore we have to consider three scenarios, namely $\lambda < a$, $\lambda \approx a$ and $\lambda > a$. We choose in all simulations the same input-wavelet ($\lambda_0 = 40$ m) and vary the correlation lengths ($a_1 = \lambda_0/4, a_2 = \lambda_0, a_3 = 4\lambda_0$) The strength of the medium fluctuations determines the spatial range of weak wavefield fluctuations, where our theory is expected to work. Reported standard deviations of the velocity in reservoir geophysics are $\sigma_v = 3 - 10\%$. We simulate the wave propagation in random media with 8% of standard deviation.

Each gray 'background' in the left-sided column of Fig. (4) consists of 50 traces (the z-component of the wavefield) recorded on a common travel-distance gather at the corresponding depths 24, 224, 424, 624 and 824 m. From the uppermost to the lowermost seismograms in Fig. (4) we clearly observe that the amplitude as well as traveltime fluctuations of traces – recorded at the same depths – increase with increasing travel-distances. This is physically reasonable since for larger travel-distances there are more interactions (scattering events) between wavefield and heterogeneities resulting in a more complex wavefield and consequently in more variable waveforms along the transverse distance relative to the main propagation direction. Finally, the thicker black curve denotes the result of convolving the analytically computed Green's function with the input-wavelet w(t).

Our wavefield description is based upon self-averaged quantities and should be able to describe the most probable primary as well as be a good approximation for primaries in typical seismograms. That is to say, we should look for the probability densities and find those seismograms whose wavefield attributes coincide with the maxima of the probability densities. To numerically demonstrate this, we have to compute the joint probability density function of the first arrival amplitudes and phases (traveltimes). Assuming statistical independence of amplitude A and traveltime t fluc-

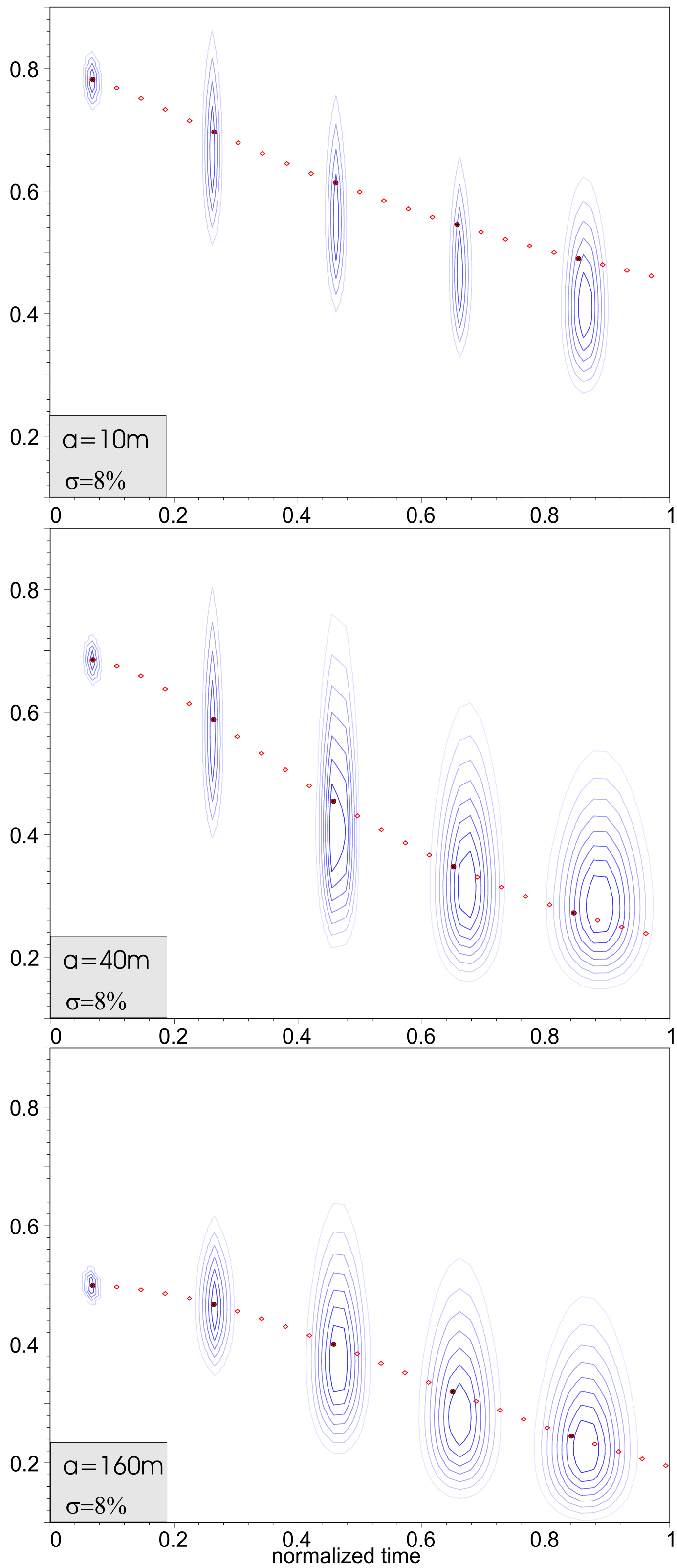

Figure 3: The contour plots show the reconstructed joint probability densities for several travel-distances.

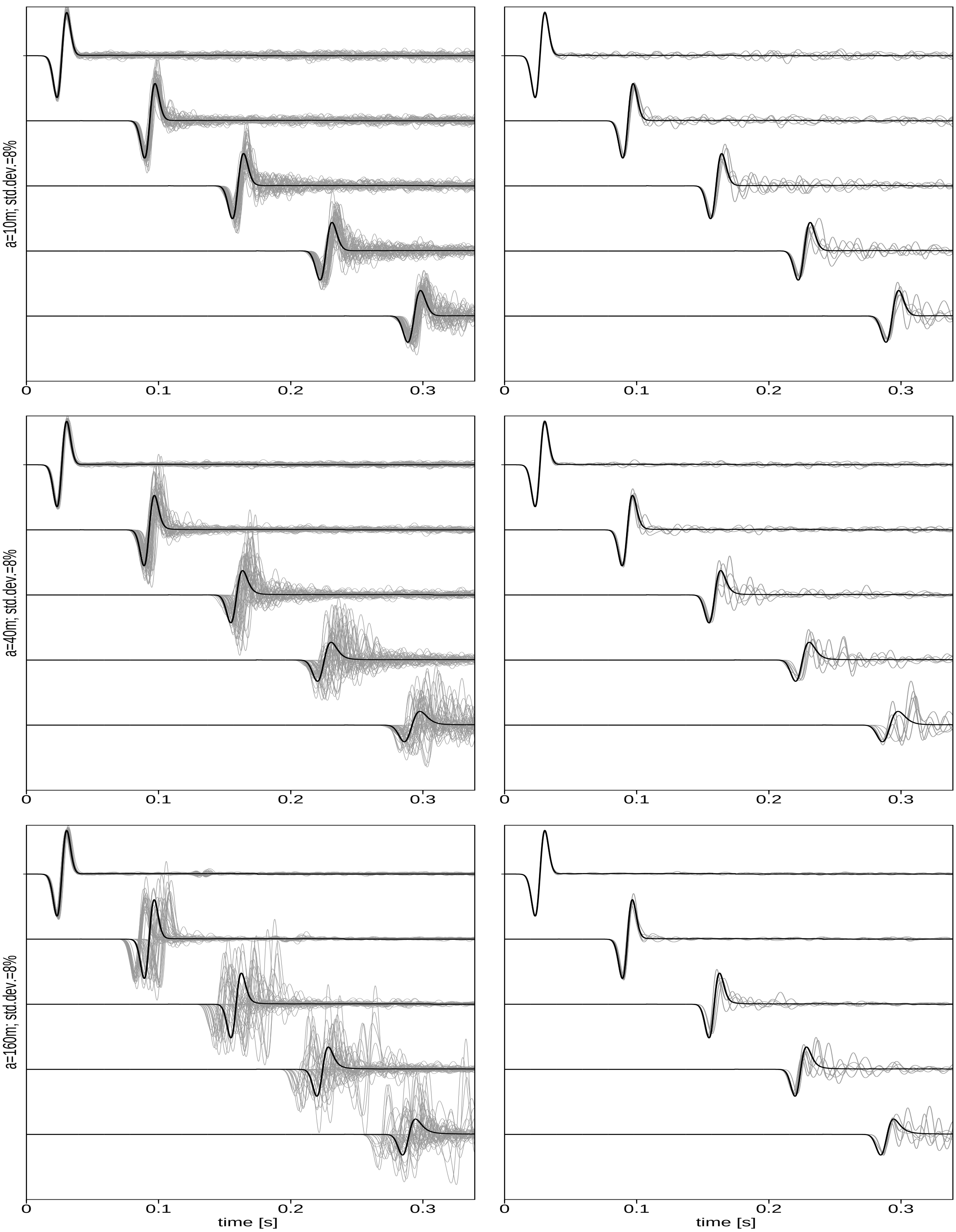

Figure 4: Waveforms of the numerical experiments compared with the theoretically predicted wavefield for $\sigma = 8\%$ and $a = 10, 40, 160m$ (from top to bottom).

tuations, we can write the joint probability density p as $p(A,t,L) = p(A,L)\,p(t,L)$. A robust estimate of $p(A,L)$ and $p(t,L)$ from the simulated data can be obtained by the maximum-likelihood method. The contour plots in Fig. (3) display the joint probability function for the travel-distances L =24, 224, 424, 624 and 824 m. On the other hand, the diamonds in Fig. (3) denote the amplitude and traveltime values as predicted by our wavefield description. The filled diamonds refer to the aforesaid travel-distances. We observe that for all travel-distances these points coincide well with the maxima of the joint probability densities.

Now, the grey seismograms in right-sided columns of Fig. (4) are selected in such a way that their amplitudes and traveltimes are typical ones (i.e., situated in the vicinity of the most probable amplitudes and traveltimes according to the maxima of the 2-D probability density functions in Fig. (3)). That is what we call typical seismograms. Of course, there are various seismograms which could be selected in this fashion (10% of the seismograms displayed on the left side). The black curves in Fig. (4) (right columns) denote the theoretically predicted wavefield based on the Green's function. We observe an good agreement between theory and experiment for the primary arrivals. Comparing left and right plots in Fig. (4) clearly demonstrates that the analytical curves give estimates of the primary wavefields for typical single traces. To summarize, with help of a statistical analysis of the recorded wavefields we can verify our theoretical results. No matter if $\lambda > a$ or $\lambda < a$, the our method is able to predict the wavefield around the primary arrivals.

CONCLUSIONS

We construct the complex, ensemble averaged wavenumber of an initially plane wave propagating in 2-D and 3-D weakly heterogeneous random media. These logarithmic wavefield attributes, obtained by combination of the Rytov approximation and the causality principle, are self-averaged quantities and allow to describe any typical, single realization of the wavefield. Typical wavefield realizations or seismograms are those that are nearly identical to the most probable realization. This approach describes the wavefield in the vicinity of the primary arrivals. The analytical results are confirmed with help of finite-difference simulations in 2-D elastic random media. A statistical analysis of the simulated wavefield shows how typical realizations of seismograms can be identified. The proposed method agrees well with numerical results.

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