Wave Inversion Technology, Report No. 4, pages 203-215

Second-order interpolation of traveltimes

C. Vanelle and D. Gajewski[1]

keywords: *traveltimes, ray method, paraxial rays, $T^2 - X^2$-method, migration*

ABSTRACT

To carry out a 3-D prestack migration of the Kirchhoff type is still a task of enormous computational effort. Its efficiency can be significantly enhanced by employing a fast traveltime interpolation algorithm. High accuracy can be achieved if second order spatial derivatives of traveltimes are taken into account to acknowledge the curvature of the wavefront. We suggest a hyperbolic traveltime interpolation scheme that allows for the determination of the hyperbolic coefficients directly from traveltimes sampled on a coarse grid, thus reducing the requirements in data storage. The approach is closely related to the paraxial ray approximation and corresponds to an extension of the well-known $T^2 - X^2$ method to arbitrary heterogeneous and complex media in 3-D. Application to various velocity models including a 3-D version of the Marmousi model confirms its superiority to the popular trilinear interpolation. This is especially the case for regions with a strong curvature of the local wavefront. Contrary to trilinear interpolation our method also provides the possibility to interpolate source positions, which is a factor 5-6 faster than the calculation of traveltime tables using a fast finite differences eikonal solver.

INTRODUCTION

Using finite difference eikonal solvers or the wavefront construction method (for an overview of both, see (Leidenfrost et al., 1999)) traveltime tables can be computed efficiently. This is one foundation for the summation stack along diffraction surfaces for a Kirchhoff type migration. For a 3-D prestack depth migration, however, tremendous amounts of traveltimes are needed: fine gridded traveltime maps are required for a vast number of sources. The need in computational time as well as in data storage can be significantly reduced by using fast and accurate traveltime interpolation routines such that only few original traveltime tables must be computed and stored on coarse grids, whereas fast interpolation is carried out onto the finer migration grid.
In 1982, Ursin introduced a hyperbolic approximation for reflection traveltimes where

[1]**email:** vanelle@dkrz.de

the wavefront curvature matrix employed is determined by dynamic ray tracing. A general second order approximation of traveltimes in seismic systems was established by (Bortfeld, 1989). His work is based on the paraxial ray approximation and can be used to interpolate traveltimes for sources and receivers which are located in the bordering surfaces of the seismic system. (Schleicher et al., 1993) link the Bortfeld theory to the ray propagator formalism and introduce a hyperbolic variant of paraxial traveltime interpolation. Both methods are, however, restricted to source and receiver reference surfaces and require the application of dynamic ray tracing. (Mendes, 2000) suggests traveltime interpolation using the Dix hyperbolic equation. Since the Dix equation is only valid for horizontally layered media, this technique is not justified for other models. (Brokesová, 1996) states the superiority of the paraxial (parabolic) interpolation compared to linear and Fourier (sinc-) interpolation of traveltimes. (Gajewski, 1998) not only finds the hyperbolic variant of paraxial approximation to be far superior to trilinear interpolation but also introduces a technique to determine the interpolation coefficients directly from traveltimes, therefore providing a means to avoid dynamic ray tracing. The algorithm is, however, restricted to horizontal interpolation. Although the procedure can be repeated for vertical receiver lines this does not allow for data reduction onto vertical coarse grids.

The method of traveltime interpolation that we present in this paper is neither restricted to laterally homogeneous media nor to interpolation in reference surfaces respectively horizontal layers only. We overcome the latter problem by introducing a technique to compute also coefficients for vertical interpolation. With all properties being determined from traveltimes only our method corresponds to an extension of the well known $T^2 - X^2$ technique to arbitrary heterogeneous media. No dynamic ray tracing is necessary.

Following the derivation of the parabolic and hyperbolic traveltime expansion in 3-D we give a detailed description of the implementation of our algorithm. Interpolation coefficients are determined from coarse gridded traveltimes including coefficients necessary for vertical interpolation. We then demonstrate the quality of our method by applying it to a variety of velocity models ranging from examples for with analytical solutions are known to a 3-D extension of the highly complex Marmousi model. We compare our results for both parabolic and hyperbolic interpolation to trilinear interpolation in accuracy and performance. We also investigate the influence of the size of the coarse grid spacing and the behaviour in the vicinity of non-smooth zones in the traveltime data. We summarize the results in the conclusions and give an outlook.

TRAVELTIME EXPANSION

The following considerations are based upon the existence of first order continuous derivatives for the velocities. For the traveltime fields continuous derivatives of first and second order are required. Traveltimes that satisfy these conditions can be expanded into a Taylor series until second degree. Provided that the distance to the expansion point is small, the Taylor series yields a good approximation for the original traveltime function. The size of the vicinity describing 'small' distances depends on the scale of velocity variations in the input model.

For the 3-D case, the Taylor expansion has to be carried out in 6 variables: the 3 components of the source position vector $\hat{s} = (s_1, s_2, s_3)^\top$ and those of the receiver position $\hat{g} = (g_1, g_2, g_3)^\top$. The values of $\hat{s}$ and $\hat{g}$ in the expansion point are $\hat{s}_0$ and $\hat{g}_0$ with the traveltime τ_0 from $\hat{s}_0$ to $\hat{g}_0$. The variations in source and receiver positions $\triangle\hat{s}$ and $\triangle\hat{g}$ are such that $\hat{s} = \hat{s}_0 + \triangle\hat{s}$ and $\hat{g} = \hat{g}_0 + \triangle\hat{g}$. The Taylor expansion for $\tau(\hat{s}, \hat{g})$ up to second order is

$$\tau(\hat{s},\hat{g}) = \tau_0 - \hat{p}_0 \triangle\hat{s} + \hat{q}_0 \triangle\hat{g} - \triangle\hat{s}^\top \hat{\mathcal{N}} \triangle\hat{g} - \frac{1}{2}\triangle\hat{s}^\top \hat{\mathcal{S}} \triangle\hat{s} + \frac{1}{2}\triangle\hat{g}^\top \hat{\mathcal{G}} \triangle\hat{g} + \mathcal{O}(3) \qquad (1)$$

with the slowness vectors at $\hat{s}_0$ and $\hat{g}_0$

$$p_{0i} = -\left.\frac{\partial\tau}{\partial s_i}\right|_{\hat{s}_0,\hat{g}_0} \qquad q_{0i} = \left.\frac{\partial\tau}{\partial g_i}\right|_{\hat{s}_0,\hat{g}_0} \quad . \qquad (2)$$

The second order derivatives are given by the matrices $\hat{\mathcal{S}}$, $\hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$ with

$$\hat{\mathcal{S}}_{ij} = -\left.\frac{\partial^2\tau}{\partial s_i \partial s_j}\right|_{\hat{s}_0,\hat{g}_0}, \quad \hat{\mathcal{G}}_{ij} = \left.\frac{\partial^2\tau}{\partial g_i \partial g_j}\right|_{\hat{s}_0,\hat{g}_0} \quad \text{and} \quad \hat{\mathcal{N}}_{ij} = -\left.\frac{\partial^2\tau}{\partial s_i \partial g_j}\right|_{\hat{s}_0,\hat{g}_0} \quad . \qquad (3)$$

Equation (1) describes the *parabolic* traveltime expansion.

Since we know that diffraction traveltimes can be expressed by hyperbolae rather then by parabolae (e.g., (Ursin, 1982), (Schleicher et al., 1993)) we will now derive a hyperbolic expression for $\tau(\hat{s}, \hat{g})$. Instead of expanding $\tau(\hat{s}, \hat{g})$ we expand its square, $\tau^2(\hat{s}, \hat{g})$, again until second order. Applying the chain rule and the abbreviations (2) and (3) leads to

$$\tau^2(\hat{s},\hat{g}) = (\tau_0 - \hat{p}_0\triangle\hat{s} + \hat{q}_0\triangle\hat{g})^2 + \tau_0 \left(-2\,\triangle\hat{s}^\top \hat{\mathcal{N}} \triangle\hat{g} - \triangle\hat{s}^\top \hat{\mathcal{S}} \triangle\hat{s} + \triangle\hat{g}^\top \hat{\mathcal{G}} \triangle\hat{g}\right) + \mathcal{O}(3) \quad . \qquad (4)$$

This equation is the *hyperbolic* traveltime expansion. The same result (4) can be obtained by squaring equation (1) and neglecting any terms of higher spatial order than two, corresponding to a Taylor expansion of (1). This approach was used by (Schleicher et al., 1993). Please note that for the derivation of (1) and (4) no assumption on the model was made. Therefore these expressions not only apply to 3-D heterogeneous media but even to anisotropic media.

A similar result for reflection traveltimes was presented by (Ursin, 1982) and (Gajewski, 1998). Gajewski considers a CMP-situation with $|\hat{s}| = -|\hat{g}| = \frac{r}{2}$ as half offset coordinate for a laterally homogeneous layered medium. Using the zero offset ray and $\hat{\mathcal{G}} = -\hat{\mathcal{S}}$ leads to

$$\tau^2 = \tau_0^2 + \frac{1}{2}\tau_0 \hat{\mathcal{N}} r^2 = \tau^2 + \frac{r^2}{v_{\text{nmo}}^2} \quad . \qquad (5)$$

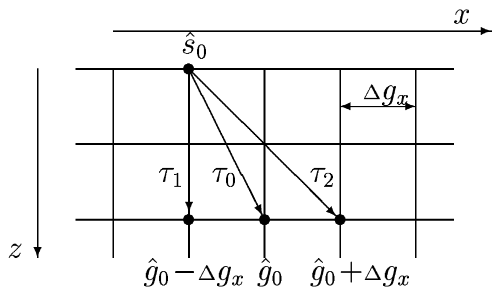

Figure 1: Determination of the coefficients q_{0x} and $\hat{\mathcal{G}}_{xx}$: traveltimes τ_0 from $\hat{s}_0$ to $\hat{g}_0$, τ_1 from $\hat{s}_0$ to $\hat{g}_0 - \Delta g_x$ and τ_2 from $\hat{s}_0$ to $\hat{g}_0 + \Delta g_x$ are required.

The move-out velocity $v_{\mathrm{nmo}} = \sqrt{2/\tau_0 \hat{\mathcal{N}}}$ is a good approximation for the RMS-velocity of the Dix formula. Therefore our technique can be considered to be an extension of the well known $T^2 - X^2$-method to arbitrary 3-D heterogeneous media.

IMPLEMENTATION

Equations (1) and (4) state that for small variations of $\Delta \hat{s}$ and $\Delta \hat{g}$ traveltimes can be interpolated with high accuracy if the according coefficient sets are known. This means that we can not only interpolate in between receivers but also in between sources. (Ursin, 1982) has presented examples for coefficients determined from ray tracing. We can, however, use (1) and (4) not only for interpolation but also for the determination of the coefficients if traveltimes for certain source-receiver combinations are given. Since we aim for migration such data are available. We will now demonstrate how to obtain the coefficients from traveltimes sampled on a coarse grid. Subsequent interpolation onto the required fine grid can then be carried out.
In the following we will refer to cartesian grids for both source and receiver positions. Sources are located in the x-y-plane with x, y and z corresponding to the indices 1,2 and 3 of the previous section. To determine the slowness vectors $\hat{p}_0$ and $\hat{q}_0$ as well as the second order derivative matrices $\hat{\mathcal{S}}$, $\hat{\mathcal{G}}$ and $\hat{\mathcal{N}}$, we need tables containing traveltimes from the source to each subsurface point for eight neighbouring sources of the source under consideration at $\hat{s}_0$ (for a 2D model the number of additional sources reduces to two). These additional sources are placed on the x-y-grid with a distance to the central source that coincides with the coarse grid spacing Δx and Δy. Please note that the method is not restricted to cubical grids. The components of $\hat{p}_0$, $\hat{\mathcal{S}}$ and $\hat{\mathcal{N}}$ carrying indices x and y only are determined directly from the traveltime tables, as are all elements of $\hat{q}_0$ and $\hat{\mathcal{G}}$.

To give an example: to compute q_{0x} and $\hat{\mathcal{G}}_{xx}$ we need only the traveltimes τ_0, τ_1 and

τ_2 as they are shown in Figure 1. We insert τ_1 and τ_2 into the parabolic expansion (1) respectively. Building the sum and the difference of the resulting expressions yields the following result:

$$q_{0x} = \frac{\tau_2 - \tau_1}{2\,\triangle g_x} \quad \text{and} \quad \hat{\mathcal{G}}_{xx} = \frac{\tau_1 + \tau_2 - 2\,\tau_0}{\triangle g_x^2} \quad . \tag{6}$$

For the hyperbolic form we find a similar solution. Inserting τ_1, τ_2 and τ_0 into (4) leads to

$$q_{0x} = \frac{\tau_2^2 - \tau_1^2}{4\,\tau_0\,\triangle g_x} \quad \text{and} \quad \hat{\mathcal{G}}_{xx} = \frac{\tau_2^2 + \tau_1^2 - 2\,\tau_0^2}{2\,\tau_0\,\triangle g_x^2} - \frac{q_{0x}^2}{\tau_0} \quad . \tag{7}$$

The y- and z-components of $\hat{q}_0$ and $\hat{\mathcal{G}}$ can be found in the same way by varying g_y respectively g_z. Varying both g_x and g_y leads to $\hat{\mathcal{G}}_{xy}$; $\hat{\mathcal{G}}_{yz}$ and $\hat{\mathcal{G}}_{zx}$ follow accordingly. The determination of the x- and y-components of $\hat{\mathcal{S}}$ and $\hat{p}_0$ is straightforward: instead of varying the receiver position we use different source positions. For the xx-, yy-, xy- and yx-components of $\hat{\mathcal{N}}$ both source and receiver positions have to be varied. But this does not yet give us the z-components. Unless we compute also traveltimes for sources at different depths – which we do not intend to – another approach is needed. At this point we make use of the eikonal equation to express the z-component of the slowness vector $\hat{p}_0$ as

$$p_{0z} = \sqrt{\frac{1}{v_s^2} - p_{0x}^2 - p_{0y}^2} \tag{8}$$

where v_s is the velocity at the source, provided that the source lies in the top surface of the model. Otherwise we have to insert a sign in (8). Since second order traveltime derivatives are also first order derivatives of slownesses we can rewrite $\hat{\mathcal{S}}$ and $\hat{\mathcal{N}}$ to

$$\hat{\mathcal{S}}_{ij} = \left(\frac{\partial p_i}{\partial s_j}\right)_{\hat{s}_0,\hat{g}_0} \quad \text{and} \quad \hat{\mathcal{N}}_{ij} = -\left(\frac{\partial q_j}{\partial s_i}\right)_{\hat{s}_0,\hat{g}_0} = \left(\frac{\partial p_i}{\partial g_j}\right)_{\hat{s}_0,\hat{g}_0} \quad . \tag{9}$$

If we now substitute p_{0z} in equation (9) by (8) we can compute the second order derivatives of τ with respect to s_z and g_z from the already known x-y-matrix elements and derivatives of the velocity. Since we assume the velocity field to be smooth, the velocity derivatives can be determined with a second order FD operator on the coarse grid. To give an example the matrix element $\hat{\mathcal{S}}_{xz}$ can be expressed by v_s, $\hat{\mathcal{S}}_{xx}$ and $\hat{\mathcal{S}}_{xy}$ as

$$\hat{\mathcal{S}}_{xz} = -\frac{1}{v_s^3\,p_{0z}}\frac{\partial v_s}{\partial s_x} - \frac{p_{0x}}{p_{0z}}\,\hat{\mathcal{S}}_{xx} - \frac{p_{0y}}{p_{0z}}\,\hat{\mathcal{S}}_{xy} \quad . \tag{10}$$

This expression will not yield a result for $\hat{\mathcal{S}}_{xz}$ if p_{0z} equals zero. This case has, however, no practical relevance for the applications that the method was developped for. The derivation of the remaining z-components of $\hat{\mathcal{S}}$ and $\hat{\mathcal{N}}$ is straightforward.

Table 1: Median relative errors for the homogeneous velocity model.

Interpolation:	original source:	shifted source:
hyperbolic:	10^{-5} %	10^{-5} %
parabolic:	0.014 %	0.023 %
trilinear:	0.401 %	not available

EXAMPLES

Our first example is a model with constant velocity. We used analytical traveltimes as input data. Therefore errors are only due to the method itself and possibly roundoff errors. The example model is a cube of 101×101×101 grid points with 10m grid spacing. The source is centered in the top surface. Input traveltimes were given on a cubical 100m coarse grid. The distances in source position were also 100m in either direction. Coefficients were computed for both hyperbolic and parabolic variants. For each variant interpolation onto a 10m fine grid was carried out twice: for the original source position and for a source shifted by 50m in x and y. Both were compared to analytic data. The resulting relative traveltime errors are displayed in Figure 2. They are summarized in Table 1 together with results for a trilinear interpolation using the 100m coarse grid traveltimes as input data. A layer of 50m depth under the source was excluded from the statistics. We find the hyperbolic interpolation superior to the parabolic variant. Both exceed the trilinear interpolation by far.

We use *median* errors, not mean errors. This is due to the stability of the median concerning outliers. Therefore, the median error is a more reliable value compared to the mean error.

The second model is again an example where the analytical solution is known. It has a constant velocity gradient of $\partial v/\partial z$=0.5s^{-1} and the velocity at the source is 3km/s. The source positions and dimensions are the same as in the first example. The results are shown in Figure 3 and in Table 2. As before, a layer of 50m depth under the source was excluded from the statistics. Again we find the hyperbolic results better than the parabolic ones and both far superior to trilinear interpolation. The difference in quality between hyperbolic and parabolic interpolation is less than for the constant velocity model. The reason is that for a homogeneous medium the hyperbolic approximation is equal to the analytic result.

The constant velocity gradient model was also used to investigate the influence of the coarse grid spacing on the accuracy. Traveltime interpolation was carried out for coarse grid spacings ranging from 20 to 100m using hyperbolic, parabolic and trilinear interpolation. The fine grid spacing remained fixed at 10m. The resulting

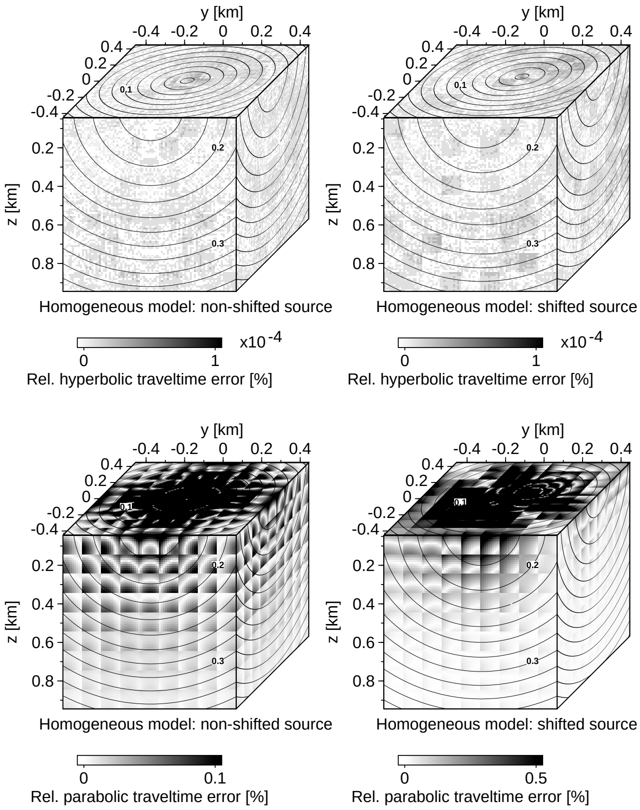

Figure 2: Relative traveltime errors for a homogeneous velocity model. Top: errors using the hyperbolic interpolation if only receivers are interpolated (left) and for both source and receiver interpolation (right). Isochrones are given in seconds. Bottom: the same as above but using the parabolic variant. The relative errors near the source appear exaggerated because there the traveltimes are very small. Please note the different error scales.

errors are displayed in Figure 4. We find the same quality relation between the three interpolation schemes as before and, as expected, the accuracy increasing for smaller coarse to fine grid ratio.

The reason for the much higher accuracy of the parabolic and hyperbolic interpolation is obviously that trilinear interpolation neglects the wavefront curvature. Unlike in the previous examples this is not only a problem in the near-source region but anywhere where we find locally higher wavefront curvatures. This is especially very common for more complex velocity models, which we consider in the following. We now apply our method to a 3-D extension of the Marmousi model (Versteeg and Grau, 1991). Input traveltimes were computed with a 3-D-FD eikonal solver using the (Vidale, 1990) algorithm. The coarse grid spacing was 125m, the fine grid was

Table 2: Median relative errors for the constant velocity gradient model.

Interpolation:	original source:	shifted source:
hyperbolic	0.002 %	0.001 %
parabolic	0.009 %	0.015 %
trilinear	0.282 %	not available

12.5m. The amount of computational time necessary to carry out the hyperbolic interpolation for one shot is 14% of the time needed by the Vidale algorithm. The parabolic interpolation is only very slightly faster, it needs 13% of the time necessary

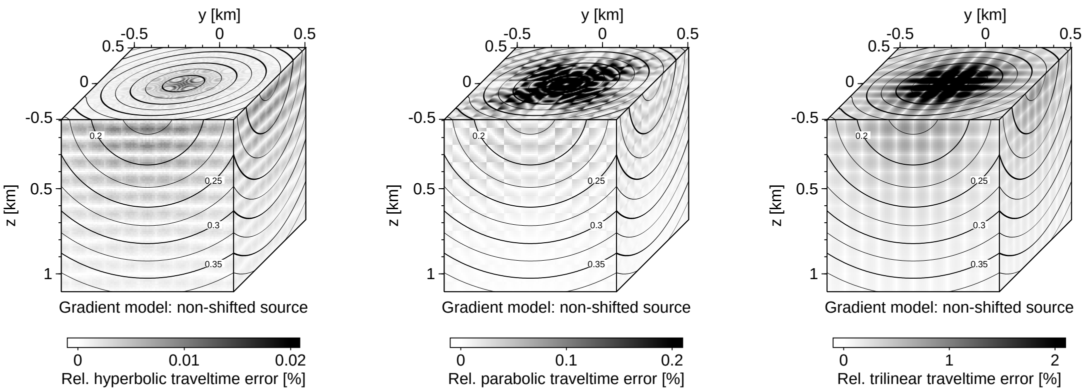

Figure 3: Relative traveltime errors for a constant velocity gradient model. Left: errors using the hyperbolic interpolation (non-shifted). Middle: the same for the parabolic interpolation. Right: as before but with trilinear interpolation. Please note the different error scales on the plots.

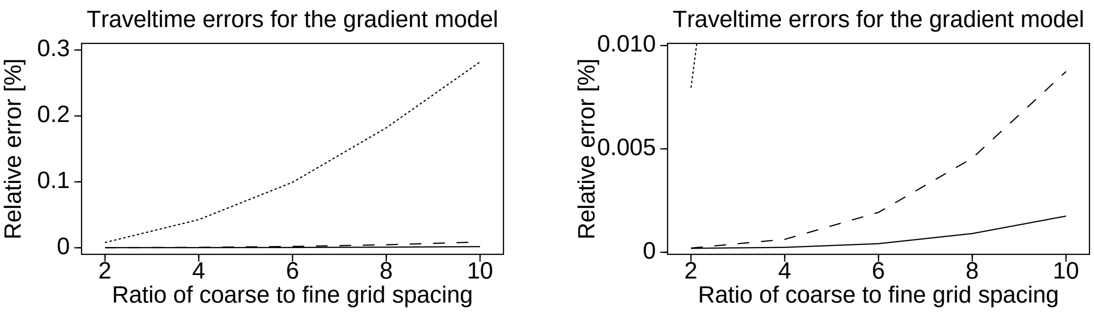

Figure 4: Relative traveltime errors vs. ratio of fine to coarse grid spacing for the constant velocity gradient model shown for trilinear (dotted line) parabolic (dashed line) and hyperbolic interpolation (solid line). The plot is displayed twice using different scales to illustrate the differences.

for Vidale. The resulting interpolated traveltimes were compared to reference data for the fine grid obtained from the same Vidale scheme. The relative traveltime errors for the hyperbolic interpolation are shown in Figure 5. A layer of 62.5m depth under the source was excluded from the statistics. We find a median error of 0.025%. The parabolic variant yields a median error of 0.026% (not shown here). Compared to the generic models both interpolations yield similar quality. The reason is that for more complex models errors due to the different algorithms are dominated by errors caused by the quality of the input data, i.e., insufficient accuracy and particularly smoothness of the input traveltimes.

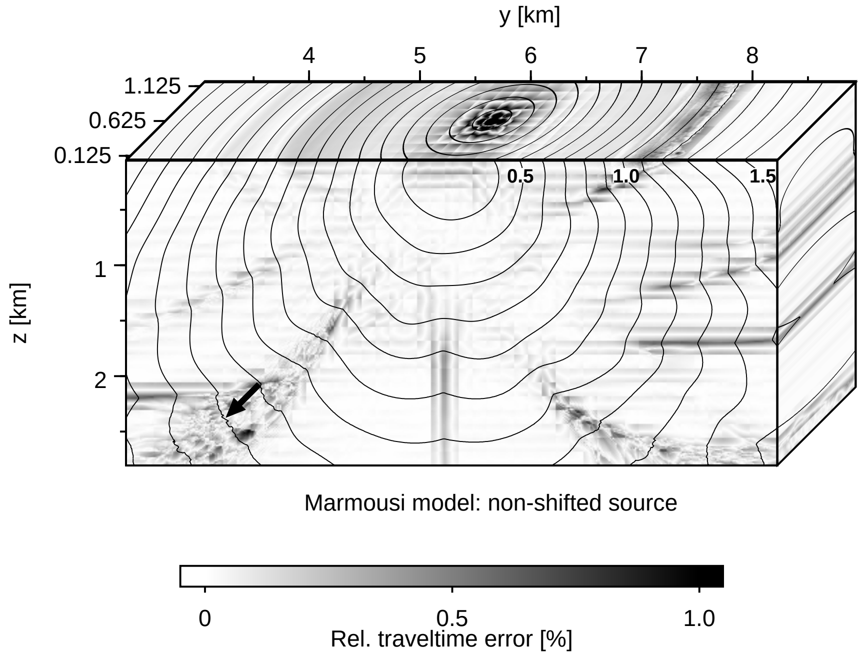

Figure 5: Relative traveltime errors for the Marmousi model using hyperbolic interpolation. Isochrones are given in seconds. The correlation of errors and 'kinks' in the isochrones is clearly visible. The arrow at the 1.2s isochrone indicates a higher error area that is caused by bad quality of the input traveltimes (due to a deficiency of the FD implementation used). This can be compensated by smoothing the input traveltimes.

In the last example we find errors in the vicinity of 'kinks' in the isochrones. These indicate triplications of the wavefronts. The resulting errors are no surprise because the assumption of smooth traveltimes does not hold anymore. The reason is that a triplication consists of wavefronts belonging to two different phases. These must be interpolated separately. The obvious solution to overcome this problem is to employ later arrivals in the input traveltime scheme and to apply our method to first and later arrivals separately.

CONCLUSIONS

We have presented a method for the interpolation of traveltimes which is based on the traveltime differences (i.e., move-out) between neighbouring sources and receivers of a multi-fold experiment. The interpolation has a high accuracy since it acknowledges the curvature of the wavefront and is thus exact to the second order. Although for complex models parabolic interpolation yields high accuracy we recommend the use of hyperbolic traveltime expansion. This commendation is supported by the generic examples. Both hyperbolic and parabolic variants are, however, far superior to the popular trilinear interpolation. The difference in computational time for the three variants is insignificant.

One important feature of our technique is its possibility to interpolate for sources, not only for receivers. The fact that all necessary coefficients can be computed on a coarse grid leads to considerable savings in time and memory since traveltime tables for less sources need to be generated as well as kept in storage. If we use, e.g., every tenth grid point in three dimensions, this corresponds to a factor of 10^6 less in storage requirement for interpolation of shots and receivers. As the method is not restricted to cubical grids the coarse grid spacing can be adapted to the model under consideration. Our method is best combined with techniques for computing traveltime tables for first and later arrivals on coarse grids, like the wavefront construction method (i.e., ray tracing). Most finite difference eikonal solvers (FDES) do not allow the computation of traveltimes on coarse grids since it reduces accuracy to an unacceptable degree. Moreover, FDES provide first arrivals only.

In the examples presented isotropic models were assumed. Since no assumptions on the models were made when deriving the governing equations (1) and (4) the technique presented here can also be applied to traveltimes tables computed for anisotropic media.

Since the matrices introduced in this paper bear a close relationship to the matrices employed in the paraxial ray approximation (cf., e.g., (Bortfeld, 1989)) our technique can also be used to determine dynamic wavefield properties. This leads to the determination of the complete ray propagator from traveltimes only which can be used for various tasks including the computation of geometrical spreading (leading to an interpolation of Green's functions) or migration weights (i.e., amplitude preserving migration) and the estimation of Fresnel zones and therefore optimization of migration apertures. Unlike for the traveltime interpolation the hyperbolic expansion is significantly better suited for these applications. A detailed discussion will be given in a follow-up paper.

ACKNOWLEDGEMENTS

We thank the members of the Applied Geophysics Group in Hamburg for continuous and helpful discussions. Special thanks go to Tim Bergmann for extensive proofreading and Radu Coman for providing a 3-D-FD eikonal solver and thus the necessary input traveltimes. This work was partially supported by the German Israeli Foundation (I-524-018.08/97), the German Research Society (DFG, Ga 350-10) and the sponsors of the Wave Inversion Technology (WIT) consortium.

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PUBLICATIONS

Previous results concerning traveltime interpolation were published by (Gajewski and Vanelle, 1999). A paper containing these results has been submitted (Vanelle and Gajewski, 2000a)