Statistical properties of reflection traveltimes in 3-D randomly inhomogeneous and anisomeric media in presence of double passage effect

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ABSTRACT

In this paper we study the statistical properties of seismic reflection traveltimes in order to characterize the inhomogeneities of the reflector overburden. Detailed analysis of these statistical properties is presented for 3-D geometry, quasi-homogeneous fluctuations of the medium parameters, anisomeric (anisotropic) fluctuations, curved rays and other. Of special importance is the double passage effect, which takes place when the pulse passes twice through one and the same inhomogeneities of the random medium, first time on its way from the source to the reflecting interface and, second time, on its way from the interface to the receiver. The effect manifests itself in doubling of traveltime variance near the source as compared to the variance at offsets larger than the horizontal correlation radius.

INTRODUCTION

Statistical characterization of rocks is of significant interest for many purposes and plays an important role in exploration seismology. Firstly, it has been recognized that large parts of the lithosphere show spatial heterogeneities on several length scales (Sato and Fehler, 1998) and therefore, deterministic Earth models have to be supplemented with statistical information on rock heterogeneity in order to describe correctly the propagation of seismic waves. Secondly, statistical information on rock heterogeneity can be helpful for petrophysical interpretations. For example, when estimating the quality factor of rocks from seismic data, it is of interest to know whether seismic attenuation has been caused either by lithological contrasts, leading to scattering attenuation, or by viscoelastic properties of rocks, leading to intrinsic attenuation. Thirdly, in combination with usual macro-model based imaging techniques, the statistical characterization of small scale heterogeneities can be used in order to retrieve 'true' reflection coefficients of large scale heterogeneities from seismic data. Moreover, the statistical characterization of rocks can contribute to the geostatistical modeling of reservoirs, to estimates of uncertainties of seismic images, to a better understanding of different features of structures and geoprocesses. The statistical characterization implies estimates for two main parameters of rocks: horizontal scale of inhomogeneities and variation of elastic wave velocity v (or its inverse value – slowness $\mu = 1/v$).

A quite promising method for estimating the characteristic horizontal length of small-scale inhomogeneities $l_{\rm hor}$ from traveltime fluctuations $\tilde{t}$ of a pulse signal, reflected backward to the source was suggested by Touati (1996) and analyzed by Iooss (1998). The method is based on the comparison of traveltime variance $\mathrm{var}[\tilde{t}(X=0)]$, measured near the source (zero offset, $X=0$) with the variance at large offsets, $X \gg l_{\rm hor}$, which somewhat conditionally can be written as $\mathrm{var}[\tilde{t}(\infty)]$. As shown by Iooss (1998) $\mathrm{var}[\tilde{t}(0)]$ happens to be twice as large as compared with $\mathrm{var}[\tilde{t}(\infty)]$:

$$\frac{\mathrm{var}[\tilde{t}(0)]}{\mathrm{var}[\tilde{t}(\infty)]} \approx 2\,. \tag{1}$$

The transition from zero offset, $X=0$, to sufficiently large offset, $X \gg l_{\rm hor}$, occurs at a spatial scale X, comparable with $l_{\rm hor}$. Therefore measurements of $\mathrm{var}[\tilde{t}(X)]$ as a function of X might be helpful for recovering the horizontal scale $l_{\rm hor}$ from experimental data. A further numerical analysis of the method of Touati and Iooss was presented in the paper of Gaerets et al. (2001).

The doubling of traveltime variance at $X=0$ – also called the double passage effect (DPE) – occurs when the wave passes twice through randomly inhomogeneous media with large scale (as compared to wavelength λ) inhomogeneities. The double passage effect was revealed earlier for phase-path fluctuations in other physical situations: for light waves, reflected from a mirror in a turbulent atmosphere and for radiowaves reflected from the ionosphere (see Kravtsov and Saichev, 1982, 1985 and references therein and also exercise I.7.6 from the textbook of Rytov et al., 1989).

Let us remind the background relations for the DPE based on geometrical optics. As shown in Fig. 1a, at zero offset $X=0$ the incident (down-going) and reflected (up-going) rays, $\mathbf{r}_d(0)$ and $\mathbf{r}_u(0)$ respectively, pass through one and the same inhomogeneities of the random medium. Therefore the fluctuations of the traveltimes $\tilde{t}_d(0)$ and $\tilde{t}_u(0)$ are equal, $\tilde{t}_d(0) = \tilde{t}_u(0)$, and the variance of the total traveltime $\tilde{t}(0) = \tilde{t}_d(0) + \tilde{t}_u(0) = 2\tilde{t}_d(0)$ is four times larger than the variance for one way passage $\tilde{t}_d(0)$:

$$\mathrm{var}[\tilde{t}(0)] = 4\mathrm{var}[\tilde{t}_d(0)]\,. \tag{2}$$

At the same time at sufficiently large offset X, $X \gg l_{\rm hor}$, down-going and up-going rays $\mathbf{r}_d(X)$ and $\mathbf{r}_u(X)$ pass through different inhomogeneities (Fig. 1b). Therefore, the cross-product of $\tilde{t}_d(X)$ and $\tilde{t}_u(X)$ in average is close to zero and as a result at $X \gg l_{\rm hor}$

$$\mathrm{var}[\tilde{t}(\infty)] = \mathrm{var}[\tilde{t}_d(X)] + \mathrm{var}[\tilde{t}_u(X)] + 2\,\mathrm{covar}[\tilde{t}_d(X)\tilde{t}_u(X)] \approx 2\mathrm{var}[\tilde{t}_d(X)]\,, \tag{3}$$

which is roughly half of $\mathrm{var}[\tilde{t}(0)]$. The relationship between these equations and the DPE was not clearly emphasized in the pioneering works of Touati (1996) and Iooss (1998), and in the

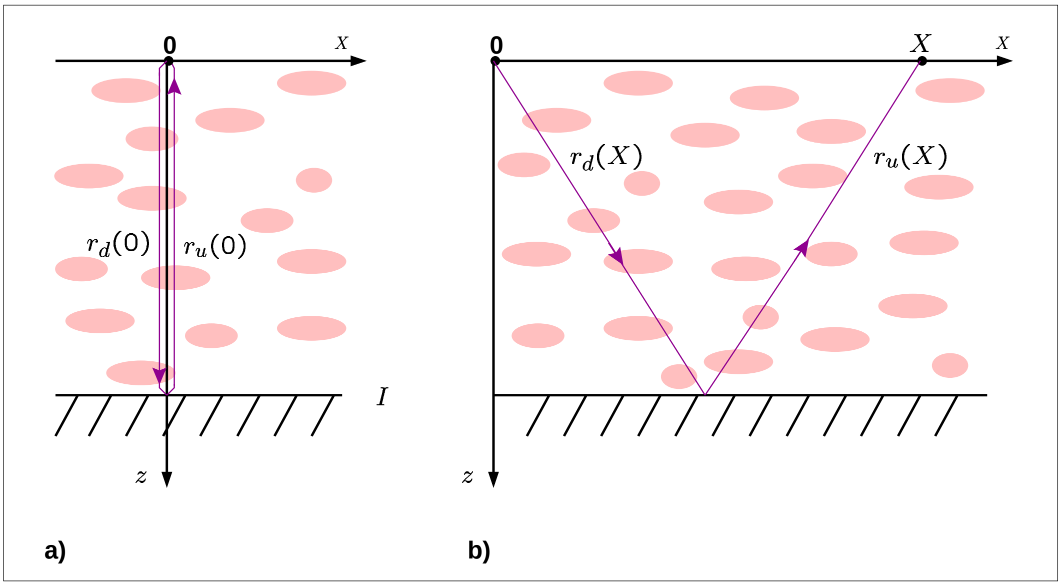


Figure 1: Geometry of the double passage effect. Fig 1a – vertical ray intersects one and the same inhomogeneities. Fig 1b – oblique ray, corresponding to large offset $X \gg l_{\text{hor}}$, passes mostly through different inhomogeneities.

successive studies by Gaerets et al. (2001).

This paper is devoted to the further analysis of DPE manifestations in seismics and to the generalization of the above mentioned results of Touati (1996), Iooss (1998) and Gaerets et al. (2001) in several directions. First of all, we describe the 3-D geometry of the DPE instead of the 2D geometry in mentioned works. Secondly, we consider fluctuations of medium parameters with quasi-homogeneous statistics. This model allows, in general, to take into account the dependence of slowness variance on depth. Thirdly, the changes of mean slowness with depth are taken into account, so the rays in our considerations might be curved. At fourth, horizontally anisomeric (anisotropic) fluctuations are considered. At fifth, our consideration deals also with covariance functions in a more general treatment than before. At last, estimates are given for the inaccuracy, characteristic for the reconstruction of the reflector position in the presence of DPE.

BASIC RELATIONS

Traveltime fluctuations in the frame of geometrical optics method

Let us consider a point source placed at $X = 0$ (see Fig. 1) radiating a pulse signal which propagates through a random medium, is reflected from a horizontal or slightly inclined plane interface I and recorded by a receiver placed in the vicinity of the source.

Supposing the inhomogeneities of the medium are large in size compared to the typical wavelength in a pulse spectrum, we apply the geometrical optics (GO) approximation for traveltime calculations. In this approximation the traveltime t in non-dispersive media obeys the formula

$$t = \int \frac{ds}{v[\mathbf{r}(s)]} = \int \mu[\mathbf{r}(s)]ds\,, \tag{4}$$

where $v(\mathbf{r})$ is the wave velocity, $\mu(\mathbf{r}) = 1/v(\mathbf{r})$ is the slowness and ds is an elementary arclength of the ray trajectory $\mathbf{r}(s)$. The last obeys the ray equations (Born and Wolf, 1999, Kravtsov and Orlov, 1990)

$$\frac{d\mathbf{r}}{ds} = \mathbf{l}\,, \qquad \frac{d\mathbf{l}}{ds} = \nabla_\perp n \equiv \nabla n - \mathbf{l}(\mathbf{l}\nabla n)\,, \tag{5}$$

where $\mathbf{l}$ is a unit vector tangent to the ray,

$$n(\mathbf{r}) = \frac{v_0}{v(\mathbf{r})} = v_0\mu(\mathbf{r}) \tag{6}$$

is the refractive index, v_0 is the wave velocity near the source and $\nabla_\perp n = \nabla n - \mathbf{l}(\mathbf{l}\nabla n)$ is a transversal (relative to the ray) gradient of the refractive index. In non-dispersive media the traveltime (4) is proportional to the eikonal ('optical path') $\Psi = \int n[\mathbf{r}(s)]ds$:

$$t = \int \mu[\mathbf{r}(s)]ds = \frac{1}{v_0}\int n[\mathbf{r}(s)]ds = \frac{\Psi}{v_0}\,. \tag{7}$$

Therefore all the results obtained earlier for the optical path fluctuations (Chernov 1960, Tatarskii, 1971, Ishimaru, 1978, Rytov et al., 1989) can be immediately used for traveltime calculations.

In a random medium slowness can be presented as a sum of regular (average), $\bar{\mu}(\mathbf{r})$, and random, $\tilde{\mu}(\mathbf{r})$ parts:

$$\mu(\mathbf{r}) = \bar{\mu}(\mathbf{r}) + \tilde{\mu}(\mathbf{r})\,, \tag{8}$$

where the mean value of $\tilde{\mu}$ is zero: $\langle\tilde{\mu}\rangle = 0$. The same is true for the ray trajectory $\mathbf{r}(s)$, traveltime $t(s)$ and the refraction index $n(s)$:

$$\mathbf{r}(s) = \bar{\mathbf{r}}(s) + \tilde{\mathbf{r}}(s), \qquad t(s) = \bar{t}(s) + \tilde{t}(s)\,, \qquad n(s) = \bar{n}(s) + \tilde{n}(s)\,. \tag{9}$$

For sufficiently weak fluctuations $\tilde{\mu}$, when

$$\sigma_\mu^2 \equiv \mathrm{var}[\tilde{\mu}(\mathbf{r})] \equiv \langle\tilde{\mu}^2(\mathbf{r})\rangle \ll 1/v_0^2\,, \tag{10}$$

and

$$\sigma_n^2 \equiv \mathrm{var}[\tilde{n}(\mathbf{r})] \ll 1\,, \tag{11}$$

the ray trajectory deviates only slightly from a regular trajectory $\bar{\mathbf{r}}(s)$ (here and henceforth both upper bar $\overline{(\cdot)}$ and angular brackets $\langle(\cdot)\rangle$ are used for statistical averaging). Therefore, as commonly accepted, first order fluctuations of traveltime $\tilde{t}$ can be calculated by integrating the perturbation $\tilde{\mu}(\mathbf{r})$ along the unperturbed ray $\bar{\mathbf{r}}(s)$ (Chernov 1960, Tatarskii, 1971, Ishimaru, 1978, Rytov et al., 1989, Snieder and Sambridge, 1992):

$$\tilde{t}(s) \cong \int \tilde{\mu}[\bar{\mathbf{r}}(s)]ds\,. \tag{12}$$

In following, we omit the upper bar over regular ray trajectory for brevity.

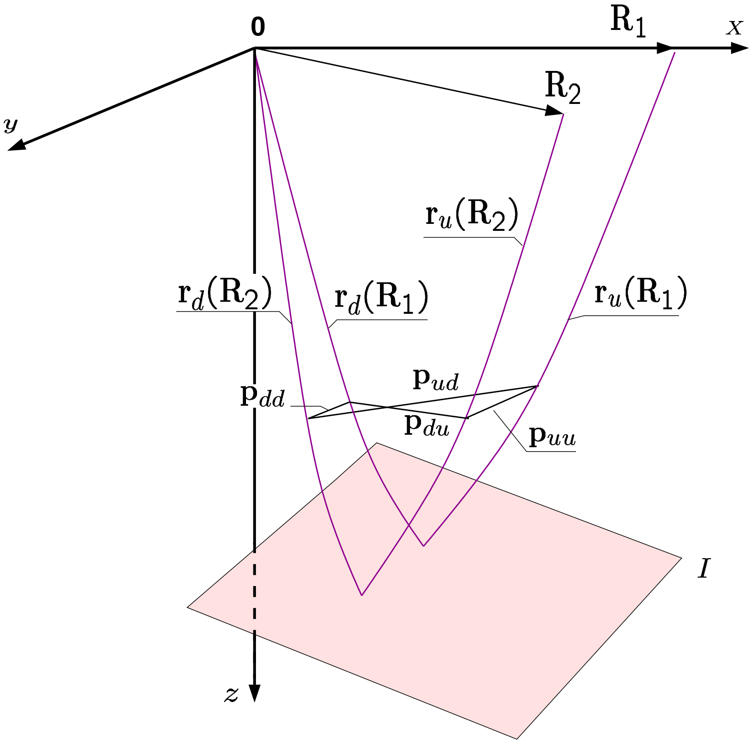


Figure 2: Two up- and down-going ray trajectories in 3-D space.

Medium fluctuations with quasi-homogeneous statistics

Quasi-homogeneous fluctuations are described by the covariance of the form (Rytov et al., 1989):

$$C_\mu(\mathbf{r}_1, \mathbf{r}_2) \equiv \mathrm{covar}[\tilde{\mu}(\mathbf{r}_1), \tilde{\mu}(\mathbf{r}_2)] \equiv \langle \tilde{\mu}(\mathbf{r}_1)\tilde{\mu}(\mathbf{r}_2)\rangle = \sigma_\mu^2(\mathbf{r}_+)K(\mathbf{r}_1 - \mathbf{r}_2; \mathbf{r}_+)\,. \tag{13}$$

Here $\mathbf{r}_+ = (\mathbf{r}_1 + \mathbf{r}_2)/2$ is the radius vector of 'center of gravity' of vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, and K is a normalized correlation function ('coefficient of correlation'), which turns out to be unit at $\mathbf{r}_1 - \mathbf{r}_2 = 0$: $K(0; \mathbf{r}_+)$=1. This quantity is supposed to decrease rapidly with difference $\mathbf{r}_1 - \mathbf{r}_2$ with a small characteristic scale l_c, but it can also slowly depend (along with σ_μ^2) on $\mathbf{r}_+$ with a large characteristic scale $l_+ \gg l_c$.

The model of quasi-homogeneous fluctuations seems to be sufficiently general and flexible for many seismological applications. On the basis of this model below we shall consider traveltime statistics for depth-dependent fluctuations of medium parameters and for anisomeric (statistically anisotropic) fluctuations.

Traveltime Covariance function

The unperturbed ray trajectory $\mathbf{r}(s)$ of a ray reflected from interface I (Fig. 2), consists of a down-going and an up-going part:

$$\mathbf{r}(s) = \begin{cases} \mathbf{r}_d(s_d;\mathbf{R}), & 0 < s_d < S_d(\mathbf{R}) \\ \mathbf{r}_u(s_u;\mathbf{R}), & 0 < s_u < S_u(\mathbf{R}) \end{cases} , \tag{14}$$

where s_d and s_u are current arclengths along down-going and up-going sections, and $S_d(\mathbf{R})$ and $S_u(\mathbf{R})$ are the corresponding total arclengths, which depend on a receiver position: $\mathbf{R} = (X, Y, 0)$ in a horizontal plane $Z = 0$. cIn equation (14) the final point $\mathbf{r}_d(S_d;\mathbf{R})$ of the down-going ray serves as the starting point of the up-going ray: $\mathbf{r}_u(s_u = 0;\mathbf{R}) = \mathbf{r}_d(S_d;\mathbf{R})$. According to equation (14), the traveltime $\tilde{t}(\mathbf{R})$ can be expressed as a sum

$$\tilde{t}(\mathbf{R}) = \tilde{t}_d(\mathbf{R}) + \tilde{t}_u(\mathbf{R}) , \tag{15}$$

where

$$\tilde{t}_d(\mathbf{R}) = \int_0^{S_d(\mathbf{R})} \tilde{\mu}[\mathbf{r}_d(s_d)]ds_d \tag{16}$$

and

$$\tilde{t}_u(\mathbf{R}) = \int_0^{S_u(\mathbf{R})} \tilde{\mu}[\mathbf{r}_u(s_u)]ds_u \tag{17}$$

are time-delay fluctuations along the down-going and up-going sections of the ray trajectory.

The covariance of traveltime, recorded by two receivers, placed at points $\mathbf{R}_1$ and $\mathbf{R}_2$, as shown in Fig. 2, has the form

$$\begin{aligned} C_t(\mathbf{R}_1,\mathbf{R}_2) &= \langle \tilde{t}(\mathbf{R}_1)\tilde{t}(\mathbf{R}_2)\rangle = \langle [\tilde{t}_d(\mathbf{R}_1) + \tilde{t}_u(\mathbf{R}_1)][\tilde{t}_d(\mathbf{R}_2) + \tilde{t}_u(\mathbf{R}_2)]\rangle \\ &= C_{dd}(\mathbf{R}_1,\mathbf{R}_2) + C_{du}(\mathbf{R}_1,\mathbf{R}_2) + C_{ud}(\mathbf{R}_1,\mathbf{R}_2) + C_{uu}(\mathbf{R}_1,\mathbf{R}_2) , \end{aligned} \tag{18}$$

where

$$\begin{aligned} C_{dd}(\mathbf{R}_1,\mathbf{R}_2) &= \langle \tilde{t}_d(\mathbf{R}_1)\tilde{t}_d(\mathbf{R}_2)\rangle & C_{ud}(\mathbf{R}_1,\mathbf{R}_2) = \langle \tilde{t}_u(\mathbf{R}_1)\tilde{t}_d(\mathbf{R}_2)\rangle \\ C_{du}(\mathbf{R}_1,\mathbf{R}_2) &= \langle \tilde{t}_d(\mathbf{R}_1)\tilde{t}_u(\mathbf{R}_2)\rangle & C_{uu}(\mathbf{R}_1,\mathbf{R}_2) = \langle \tilde{t}_u(\mathbf{R}_1)\tilde{t}_u(\mathbf{R}_2)\rangle . \end{aligned} \tag{19}$$

According to equations (16) (17) and (13), all the values (19) can be expressed in the form of double integrals over correlation function C_μ of slowness fluctuations along the non-perturbed ray trajectories. For example, the covariance C_{dd} has the form

$$\begin{aligned} C_{dd}(\mathbf{R}_1,\mathbf{R}_2) &= \int_0^{S_d(\mathbf{R}_1)} ds'_d \int_0^{S_d(\mathbf{R}_2)} ds''_d \, C_\mu[\mathbf{r}_d(s'_d;\mathbf{R}_1),\mathbf{r}_d(s''_d;\mathbf{R}_2)] \\ &= \int_0^{S_d(\mathbf{R}_1)} ds'_d \int_0^{S_d(\mathbf{R}_2)} ds''_d \, \sigma^2_\mu(\mathbf{r}_{dd+}) \, K[\mathbf{r}_d(s'_d,\mathbf{R}_1) - \mathbf{r}_d(s''_d,\mathbf{R}_2);\mathbf{r}_{dd+}] \end{aligned} , \tag{20}$$

where $\mathbf{r}_{dd+} = [\mathbf{r}_d(s',\mathbf{R}_1) + \mathbf{r}_d(s'',\mathbf{R}_2)]/2$. In what follows we analyze and simplify the expression (20) as well as the analogous expressions for C_{du}, C_{ud} and C_{uu}.

TRAVELTIME COVARIANCE FUNCTION FOR SMALL OFFSETS

Ray trajectories for small offsets

We suppose that the interface I is horizontal and that the regular and statistical properties of a medium depend only on depth z. In the simplest case when the horizontal offset vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ are small as compared to the depth D of the interface I,

$$|\mathbf{R}_{1,2}| \ll D\,, \tag{21}$$

the angles between the rays and the vertical axis z are also sufficiently small. In this case the rays are only slightly curved, so that the ray trajectories can be approximated by straight lines and the ray lengths $S_d(\mathbf{R}_1)$ and $S_u(\mathbf{R}_2)$ are equal each other and practically coincide with depth D:

$$S(\mathbf{R}_{1,2}) = \sqrt{D^2 + (R_{1,2}/2)^2} \approx D(1 + R_{1,2}^2/8D^2) \approx D\,. \tag{22}$$

Under these conditions the piece-wise rectilinear ray trajectory acquires a form:

$$\begin{aligned} \mathbf{r}_d(s_d;\mathbf{R}) &= s_d\,\mathbf{l}_d(\mathbf{R}), \quad 0 < s_d < D\,, \\ \mathbf{r}_u(s_u;\mathbf{R}) &= D\,\mathbf{l}_d(\mathbf{R}) + s_u\,\mathbf{l}_u(\mathbf{R}), \quad 0 < s_u < D\,. \end{aligned} \tag{23}$$

Here, $\mathbf{l}_d(\mathbf{R})$ and $\mathbf{l}_u(\mathbf{R})$ are the unit vectors, tangent to the down-going and up-going rays, respectively. In small offset (angle) approximation

$$\mathbf{l}_d(\mathbf{R}) \cong \frac{\mathbf{R}}{2D} + \mathbf{i}_z, \qquad \mathbf{l}_u(\mathbf{R}) \cong \frac{\mathbf{R}}{2D} - \mathbf{i}_z\,, \tag{24}$$

where $\mathbf{i}_z$ is the unit vector in z-direction.

Traveltime covariance for down-going and up-going rays

Let us introduce new variables into equation (20)

$$\xi = s'_d - s''_d\,, \qquad \zeta = (s'_d + s''_d)/2\,, \tag{25}$$

and let us expand trajectories $\mathbf{r}_d(s'_d;\mathbf{R}_1)$ and $\mathbf{r}_d(s''_d;\mathbf{R}_2)$ into power series in difference variable ξ, saving only the first order term in ξ in a difference $\mathbf{r}_d(s'_d;\mathbf{R}_1) - \mathbf{r}_d(s''_d;\mathbf{R}_2)$ and only the zeroth-order term in $\mathbf{r}_{dd+}$, which in fact happens to be $\mathbf{i}_z\zeta$.

Due to fast decrease of the correlation coefficient K with $\mathbf{r}_1 - \mathbf{r}_2$, one can extend the limits of integration in ξ to infinity and take the least value of $S(\mathbf{R}_1)$ and $S(\mathbf{R}_2)$ as upper limit in the ζ variable, as it is commonly used in statistical theory of wave propagation through random media (Chernov, 1960, Rytov et al., 1989). In view of (22) $\min[S(\mathbf{R}_1), S(\mathbf{R}_2)] \approx D$. As a result

$$C_{dd}(\mathbf{R}_1,\mathbf{R}_2) = 2\int_0^D d\zeta \quad \sigma_\mu^2(\mathbf{i}_z\zeta) \int_0^\infty d\xi\, K[\mathbf{i}_z\xi + \mathbf{p}_{dd}(\zeta); \mathbf{i}_z\zeta]\,. \tag{26}$$

Here

$$\mathbf{p}_{dd}(\zeta) = \frac{\mathbf{p}\zeta}{2D}, \qquad \mathbf{p} = \mathbf{R}_1 - \mathbf{R}_2, \tag{27}$$

is the horizontal distance between rays at their down-going sections and $\mathbf{i}_z\zeta$ stands for the center of gravity radius vector $\mathbf{r}_{dd+}$. The factor 2 in (26) arises because the integral over ξ from $-\infty$ to ∞ of the even function $K(\mathbf{r}_1 - \mathbf{r}_2)$ can be presented as doubled integral from 0 to ∞.

A formula, quite similar to (26), can also be obtained for the traveltime covariance of the up-going part of the ray trajectory:

$$C_{uu}(\mathbf{R}_1, \mathbf{R}_2) = 2\int_0^D d\zeta \quad \sigma_\mu^2(\mathbf{i}_z\zeta) \int_0^\infty d\xi\, K[\mathbf{i}_z\xi + \mathbf{p}_{uu}(\zeta); \mathbf{i}_z\zeta]\,. \tag{28}$$

Here

$$\mathbf{p}_{uu}(\zeta) = \mathbf{p}(1 - \zeta/2D) \tag{29}$$

stands for a horizontal distance between up-going rays at the level ζ. This difference equals to $\mathbf{p} = \mathbf{R}_2 - \mathbf{R}_2$ at the surface of observation, where $\zeta = 0$ and reduces to $\mathbf{p}/2$ at the reflector $\zeta = D$.

Cross-covariances and total traveltime covariance

Somewhat more complex expressions take place for cross-covariances C_{du} and C_{ud}. In fact, they are given by formulas similar to (26) and (28) only with other horizontal distances between rays:

$$\mathbf{p}_{du}(\zeta) = \mathbf{R}_2 + (\mathbf{p}/2 - \mathbf{R}_2)\frac{\zeta}{D} = \mathbf{R}_2 - \frac{\zeta}{D}\mathbf{R}_+, \tag{30}$$

$$\mathbf{p}_{ud}(\zeta) = \mathbf{R}_1 - (\mathbf{p}/2 + \mathbf{R}_1)\frac{\zeta}{D} = \mathbf{R}_1 - \frac{\zeta}{D}\mathbf{R}_+, \tag{31}$$

where $\mathbf{R}_+ = (\mathbf{R}_1 + \mathbf{R}_2)/2$. All the differences between the rays $\mathbf{p}_{dd}, \mathbf{p}_{uu}, \mathbf{p}_{du}$ and $\mathbf{p}_{ud}$ are schematically presented in Fig. 2.

The total covariance of traveltime is of the form

$$\begin{aligned} C_t(\mathbf{R}_1, \mathbf{R}_2) \;=\; 2\int_0^D d\zeta \quad \sigma_\mu^2(\mathbf{i}_z\zeta) \int_0^\infty d\xi\, \{K[\mathbf{i}_z\xi + \mathbf{p}_{dd}(\zeta)] + K[\mathbf{i}_z\xi + \mathbf{p}_{uu}(\zeta)] + \\ + K[\mathbf{i}_z\xi + \mathbf{p}_{du}(\zeta)] + K[\mathbf{i}_z\xi + \mathbf{p}_{ud}(\zeta)]\}\,. \end{aligned} \tag{32}$$

TRAVELTIME VARIANCES

Traveltime variance for small offsets

Assuming $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}$ in (32) and taking into account that in this case $\mathbf{p}$, $\mathbf{p}_{dd}$ and $\mathbf{p}_{uu}$ vanish and that

$$\mathbf{p}_{du}(\zeta) = \mathbf{p}_{ud}(\zeta) = \mathbf{R}(1 - \zeta/D)\,, \tag{33}$$

one can get the expression

$$\begin{aligned} \mathrm{var}[\tilde{t}(\mathbf{R})] \equiv \sigma_t^2(\mathbf{R}) &= C_t(\mathbf{R},\mathbf{R}) \qquad (34) \\ &= 4\int_0^D d\zeta\, \sigma_\mu^2(\mathbf{i}_z\zeta) \int_0^\infty d\xi \quad [K(\mathbf{i}_z\xi) + K(\mathbf{i}_z\xi + \mathbf{R}(1-\zeta/D))]. \end{aligned}$$

In the case of a constant variance σ_μ^2 this expression is equivalent to the result of Iooss (1998) for the 2-D problem and is analogous to the formulas for eikonal fluctuations under double passage phenomena (Kravtsov and Saichev, 1985). At $R = 0$, when traveltime is measured directly near the source, the traveltime variance equals

$$\mathrm{var}[\tilde{t}(0)] = 8\int_0^D d\zeta \int_0^\infty d\xi \quad \sigma_\mu^2(\mathbf{i}_z\zeta)\, K(\mathbf{i}_z\xi)\,, \qquad (35)$$

or, in the case $\sigma_\mu^2 = \mathrm{const}$,

$$\mathrm{var}[\tilde{t}(0)] = 8D\sigma_\mu^2 l_z\,. \qquad (36)$$

Here l_z is the vertical radius of correlation, defined as

$$l_z = \int_0^\infty d\xi K(\mathbf{i}_z\xi)\,. \qquad (37)$$

It is worth to remind that variance (35) is four times larger than the one-way traveltime variances, like (2) and twice as large as the variance at large offset $|\mathbf{R}| \gg l_{\mathrm{hor}}$, like (1).

Asymptotic behavior of the traveltime variance at large offset

The second term in (34) which in fact is the doubled covariance $C_{du}(\mathbf{R})$ is close to the first one, that is to $2C_{dd}(0)$ as long as the offset R is small as compared to the horizontal correlation radius l_{hor}:

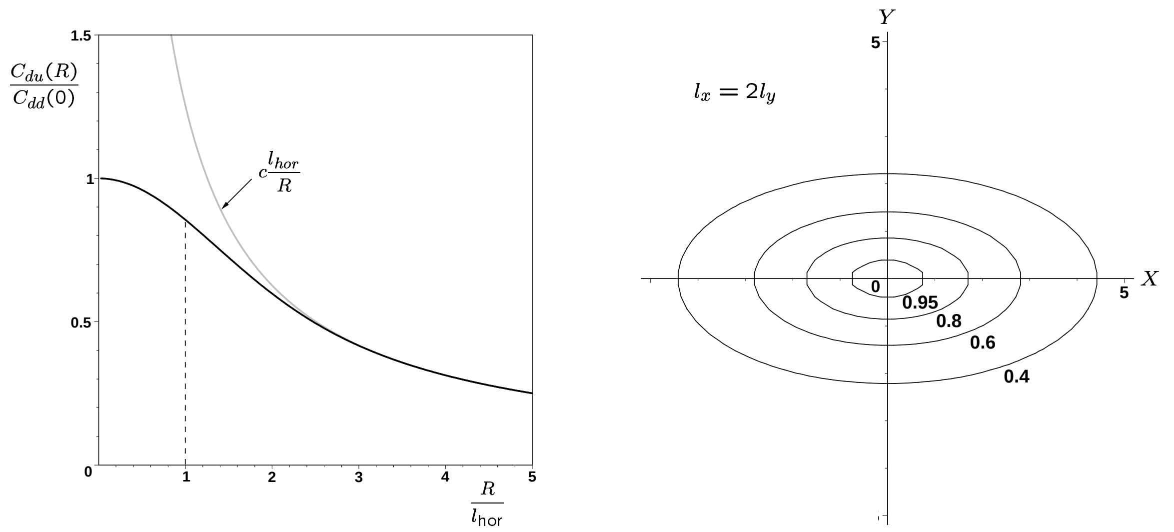
$$R < l_{\mathrm{hor}}\,. \qquad (38)$$

If $R > l_{\mathrm{hor}}$, the value $K(\mathbf{i}_z\xi + \mathbf{R}(1-\zeta/D))$ can be neglected at $\zeta = 0$, but at the same time it becomes comparable to $K(\mathbf{i}_z\xi)$ when the distance $\mathbf{p}_{du} = \mathbf{R}(1-\zeta/D)$ between down- and up-going rays is less than l_{hor}. It occurs at a critical distance $(D-\zeta)_c = l_{\mathrm{hor}}D/R$ from the reflecting surface $z = D$. Therefore the ratio $\gamma(\mathbf{R}) = C_{du}(\mathbf{R})/C_{dd}(0)$ can be estimated as the ratio of this critical distance $(D-\zeta)_c$ to total depth D:

$$\gamma(\mathbf{R}) \equiv \frac{C_{du}(\mathbf{R})}{C_{dd}(0)} \approx \frac{(D-\zeta)_c}{D} \approx \frac{l_{\mathrm{hor}}}{R}\,. \qquad (39)$$

These qualitative estimates can be supported by analytical calculations for the Gaussian correlation function

$$K(\mathbf{r}_1 - \mathbf{r}_2) = \exp\left(-\frac{(x_1-x_2)^2}{l_x^2} - \frac{(y_1-y_2)^2}{l_y^2} - \frac{(z_1-z_2)^2}{l_z^2}\right)\,. \qquad (40)$$



(a) **(b)**

Figure 3: Asymptotic behavior of the traveltime variance for sufficiently large offsets$R \gg l_x$. The ratio $C_{du}(\mathbf{R})/C_{dd}(0)$ and its asymptotic behavior (39) are schematically presented in (a). The normalized cross-variance $C_{du}(\mathbf{R})/C_{dd}(0)$ as a function of vector $\mathbf{R}$ for a medium characterized by anisomeric fluctuations, (b). Note that in (b) the quantities X, Y have arbitrary length units.

Asymptotics of the ratio $\gamma(\mathbf{R}) = C_{du}(\mathbf{R})/C_{dd}(0)$ in this case take the form $\gamma(\mathbf{R}) \approx cl_{\text{hor}}/R$, with $c \approx \sqrt{\pi}/2$. A plot of the ratio $\gamma(\mathbf{R})$ for the case $l_x = l_y = l_{\text{hor}}$ is presented by the black curve in Fig. (3(a)). A grey line on the same figure corresponds to the asymptotic behavior $\gamma(\mathbf{R}) \cong cl_{\text{hor}}/R$. It is worth to note that the analysis of the asymptotics of the ratio $\gamma(\mathbf{R})$ at $R \gg l_{\text{hor}}$ can serve as one more method to estimate horizontal correlation length l_{hor} from experimental data, additional to the straightforward estimate of l_{hor} from the plot of $\sigma_+^2(R)$ (Iooss, 1998, Gaerets et al., 2001).

Horizontally anisomeric inhomogeneities

Expression (34) enables to consider anisomeric fluctuations, which are characterized by different correlation lengths, say l_x and l_y, for different horizontal directions. Traditionally such fluctuations are referred to as anisotropic ones, though spatial scales l_x and l_y are not connected with the real anisotropy of an elastic medium. Here, the terminology 'anisomeric' fluctuation might be convenient and not confusing.

Let the statistical properties of the elastic medium be described by the anisomeric Gaussian correlation function (40). Fig. 3(b) presents the normalized cross-variance $\gamma(\mathbf{R}) = C_{du}(\mathbf{R})/C_{dd}(\mathbf{R})$ as a function of the 2D vector $\mathbf{R} = (X, Y)$ for the case $l_x = 2l_y$. The function $\gamma(\mathbf{R})$ has different spatial scales in X and Y direction, which are proportional to correlation lengths l_x and l_y.

INFLUENCE OF DPE ON THE ACCURACY OF REFLECTOR DEPTH ESTIMATION

Estimate of inaccuracy in the frame of LMS method

As revealed first by Touati (1996) and Iooss (1998), the DPE represents new opportunities to reconstruct the horizontal scales of random inhomogeneities. However, doubling of the traveltime variance at zero offset also reduces the accuracy of reflector depth estimation. In this section we consider the influence of random inhomogeneities on the accuracy of the interface reconstruction. In our simplified analysis we ignore other sources of inaccuracy (noise in receivers, regular changes of sound velocity along x, y and z directions, irregular form of interface and so on) and concentrate only on influence of the DPE in the very simple geometry: constant mean velocity v_0, homogeneous fluctuations of medium parameters, near offsets.

The traveltime t_i can be presented as

$$t_i = \frac{2}{v_0}\sqrt{D^2 + \left(\frac{x_i}{2}\right)^2} + \tilde{t}_i\,, \tag{41}$$

or, at $(x/2) \ll D$, as

$$t_i \cong \frac{1}{v_0}(2D + \frac{x_i^2}{4D}) + \tilde{t}_i\,. \tag{42}$$

Here, $\tilde{t}_i$ are traveltime fluctuations due to random inhomogeneities. Denoting the estimate for depth D as D_e, and assuming that the difference $\delta' = D_e - D$ is small enough (due to the weak fluctuations), we estimate a regular traveltime from the source to point x_i as

$$t_{ie} \equiv t_e(x_i) = \frac{1}{v_0}(2D_e + \frac{x_i^2}{4D_e}) \approx \frac{1}{v_0}\left[2D + \frac{x_i^2}{4D} + \delta(2 - \frac{x_i^2}{4D^2})\right]\,. \tag{43}$$

The correction $\delta = D_e - D$ can be defined by least squares method, that is from requirement:

$$G = \sum_{i=1}^{N}(t_i - t_{ie})^2 = \min\,, \tag{44}$$

where N stands for the total amount of receivers. The condition $\partial G/\partial\delta = 0$ minimizes the error for G and leads to the following equation for δ:

$$\sum_{i=0}^{N-1}\left[M_i\tilde{t}_i - M_i^2\frac{\delta}{v_0}\right] = 0\,, \qquad M(x_i) = 2 - \frac{x_i^2}{4D^2}\,. \tag{45}$$

Its solution

$$\delta = v_0\frac{\sum_{i=1}^{N} M_i\tilde{t}_i}{\sum_{i=1}^{N} M_i^2} \tag{46}$$

expresses the inaccuracy of the estimate D_e via traveltime fluctuations $\tilde{t}_i$. For moderate offsets $x_i \leq x_{\max} = D/4$, where the small offset approximation still works, the factors M_i are close to unit. Say at largest offset $x_{\max} = D/4$ all the values M_i lay in the interval 1 (at $x_i = 0$) and 0.94 (at $x_i = D/4$). Therefore further calculations we perform for $M_i = 1$, which results in

$$\delta = \frac{v_0}{N} \sum_{i=0}^{N-1} \tilde{t}_i \,. \tag{47}$$

The variance of the inaccuracy δ is given by the expression

$$\operatorname{var} \delta \equiv \sigma_\delta^2 = \frac{v_0^2}{N^2} \sum_{j=1}^{N-1} \sum_{i=1}^{N-1} C_t(x_i, x_j) \,, \tag{48}$$

which follows from (47). Here the traveltime covariances $C_t(x_i, x_j)$ refer to a receiver line the x-direction. Below, we analyze special cases of (48).

Uncorrelated traveltime fluctuations

If receivers are separated by a distance $\Delta x = x_{i+1} - x_i$, large as compared to the horizontal characteristic length l_x, $\Delta x \gg l_x$, all the cross-terms in (48) are close to zero, so that (48) takes the form

$$\sigma_\delta^2 = \frac{v_0^2}{N^2} \sum_{i=0}^{N-1} \sigma_t^2(x_i) \,. \tag{49}$$

Here, $\sigma_t^2(x_i) = C_t(x_i, x_i)$. For small offsets all values $\sigma_t^2(x_i)$, $i > 0$ are equal to each other: $\sigma_t^2(x_i) = \mathrm{const}$. Conditionally we name this constant value as $\sigma_t^2(\infty)$ (see Introduction). In the same time the term $\sigma_t^2(0)$ is twice as large: $\sigma_t^2(0) = 2\sigma_t^2(\infty)$. In this case one can rewrite the sum (49) as

$$\sigma_\delta^2 = v_0^2 \sigma_t^2(\infty) \frac{1}{N} \left(1 + \frac{1}{N}\right) \,, \tag{50}$$

where the factor $\frac{1}{N}(1 + \frac{1}{N})$ stands instead of traditional factor $1/N$, characterizing the gain in accuracy with growth of summands in (47). Thus doubling of the variance $\sigma_t^2(0)$ as compared to $\sigma_t^2(\infty)$ leads to insignificant increase of the resulting inaccuracy σ_δ as compared to the traditional $1/N$ law.

Correlated traveltime fluctuations

In this section we intend to show, that choosing the interval Δx between receivers less than horizontal correlation length l_x does not lead to an improvement of the accuracy as compared to (50). It becomes evident from the limit case of fluctuations $\tilde{t}_i$, which are completely correlated within the interval $X = N\Delta x$, small as compared to correlation length l_x. In this case all the N values in (47) are equal each other and resulting inaccuracy

$$\delta = \frac{v_0}{N} \sum_{i=0}^{N-1} \tilde{t}_i = \frac{v_0}{N} N \tilde{t}(0) = v_0 \tilde{t}(0) \tag{51}$$

is the same as for a single receiver. Grouping all the receivers from a sufficiently large receiving system into correlated groups, not exceeding the correlation scale l_x in length, one can arrive to expression similar to (50) only with a number of correlated group $M = X/l_x$. Thus, enlarging amount of receivers N above M we can not count on reduced inaccuracy σ_δ. For $X = 2km$ and $l_x \approx 100 - 400m$ (these are typical correlation lengths, used in Touati, 1996, Iooss, 1998, and Gaerets et al., 2001) maximal number of receivers should not be larger than 5 – 20. Corresponding gain $1/\sqrt{N}$ will not be larger than 2.3 – 4.4. It means that joint processing of data, obtained from a large set of receivers, practically can provide inaccuracy which is only 2-4 times less than inaccuracy of a single receiver.

CONCLUSIONS

In this paper we study the statistical properties of seismic reflection traveltimes in order to characterize the inhomogeneities of the reflector overburden. Detailed analysis of these statistical properties is presented for 3-D geometry, quasi-homogeneous fluctuations of the medium parameters, curved rays, etc.. In a forthcoming paper we will substantiate the results presented above with help of numerical experiments.

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