Traveltime-based true-amplitude migration of PS converted waves

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ABSTRACT

We propose a new strategy for true-amplitude migration of PS converted waves. Supplementary to PP imaging for AVO analysis this provides additional constraints for the shear properties of the target layer. Our method needs only coarse gridded traveltime information as input data. The coefficients for a fast and accurate traveltime interpolation algorithm and the weight functions are computed on-the-fly. *This makes the method highly efficient in terms of computational time, and particularly storage. A synthetic example illustrates the method.*

INTRODUCTION

One of the main goals of true-amplitude migration is the study of AVO behavior to investigate shear properties. PP imaging, however, fails in the presence of gas clouds. Also, to examine, e.g. the saturation of gas sands, it does not yield sufficient information (Castagna and Backus, 1993). Therefore processing of PS data has become a tool for looking through gas clouds. The use of techniques like weighted stacking inversion (Smith and Gidlow, 1987) leads to approximated PP and PS reflection amplitudes. We introduce a new strategy for true-amplitude migration of PS waves, that is an extension of Schleicher et al.'s (1993) method to converted waves. As we have shown previously for amplitude preserving migration of PP waves (Vanelle and Gajewski, 2001c), all required quantities can be determined from traveltimes on coarse grids. This makes the algorithm highly efficient in terms of computational time and storage.

METHOD

A diffraction stack of the form (Schleicher et al., 1993)

$$V(M) = -\frac{1}{2\pi} \iint\limits_{A} \mathrm{d}\xi_1\, \mathrm{d}\xi_2\, W(\xi_1, \xi_2, M)\ \left.\frac{\partial U(\xi_1, \xi_2, t)}{\partial t}\right|_{\tau_D(\xi_1,\xi_2,M)} \tag{1}$$

yields a true-amplitude migrated trace if proper weight functions $W(\xi_1, \xi_2, M)$ are applied. In equation (1) A is the aperture of the experiment (assumed to provide sufficient illumination). $\partial U(\xi_1, \xi_2, t)/\partial t|_{\tau_D(\xi_1,\xi_2,M)}$ is the time derivative of the magnitude of the displacement vector $\mathbf{U}$ in trace coordinates (ξ_1, ξ_2), that describe the source and receiver location, taken at the diffraction traveltime τ_D for a diffractor at a subsurface point M. Schleicher et al. (1993) derive the following expression for the weight functions:

$$W(\xi_1, \xi_2, M) = \mathcal{L}\sqrt{|\det \underline{\mathsf{H}}_F|}\, \mathrm{e}^{i\frac{\pi}{2}\left(1-\frac{\mathrm{sgn}\underline{\mathsf{H}}_F}{2}\right)} \quad . \tag{2}$$

In equation (2) the quantity $\mathcal{L}$ is the geometrical spreading. The matrix $\underline{\mathsf{H}}_F$ is the Hessian matrix of the difference $\tau_F = \tau_D - \tau_R$ between diffraction and reflection traveltime at the stationary point ξ_1^*, ξ_2^*, where $\vec{\nabla}\tau_F = 0$. This means that in this point the diffraction and reflection traveltime curves are tangent to each other. $\underline{\mathsf{H}}_F$ and $\mathcal{L}$ can be expressed in terms of second-order spatial derivatives of traveltimes. Please note, that equation (2) is a generic formulation of the weight functions applicable to monotypic waves and converted waves. It even holds for anisotropic media if appropriate expressions for $\mathcal{L}$ and $\underline{\mathsf{H}}_F$ are used. For PS and SS waves in anisotropic media, however, the situation is more complicated, because shear wave coupling has to be considered.

Provided that a traveltime curve is locally smooth and single-valued, τ can be expanded into a Taylor series. This corresponds to the paraxial approximation

$$\tau(\mathbf{s}, \mathbf{g}) = \tau_0 + \mathbf{q}_0 \,\Delta\mathbf{g} - \mathbf{p}_0 \,\Delta\mathbf{s} - \Delta\mathbf{s}^\top \underline{\mathsf{N}}\, \Delta\mathbf{g} + \frac{1}{2}\,\Delta\mathbf{g}^\top \underline{\mathsf{G}}\,\Delta\mathbf{g} - \frac{1}{2}\,\Delta\mathbf{s}^\top \underline{\mathsf{S}}\,\Delta\mathbf{s} \quad . \tag{3}$$

The approach is the same for multi-valued traveltimes, but the different branches of the traveltime curve have to be treated separately. In equation (3) $\mathbf{p}_0$ and $\mathbf{q}_0$ are the slowness vectors at the source ($\mathbf{s}_0 = \mathbf{s}(\tau_0)$) and receiver coordinates ($\mathbf{g}_0 = \mathbf{g}(\tau_0)$). The matrices $\underline{\mathsf{G}}$, $\underline{\mathsf{S}}$ and $\underline{\mathsf{N}}$ are the second-order derivatives of the traveltime with respect to receiver, source and mixed coordinates. Vectors and matrices have dimension two and represent a projection onto reference surfaces, e.g., the reflector (for further details, see Bortfeld 1989). Multi-fold traveltime tables are available, since they are required for the diffraction time surface anyway. Traveltimes for certain source-receiver combinations are inserted into (3), which can then be solved for the corresponding slowness components and matrix elements. If traveltime data is not directly available in the reflector surface, the coefficients can nevertheless be determined by using a form of (3) that is not restricted to a projection surface (Vanelle and Gajewski, 2001c).

Diffraction traveltimes can be decomposed into the downgoing and upgoing ray segments, each of them written in terms of (3) using appropriate labels to distinguish between both segments, e.g., by the indices 1 for the downgoing and 2 for the upgoing raypath. Expressing also the reflection traveltimes by equation (3) leads to the following result for $\underline{\mathsf{H}}_F$ (Schleicher et al., 1993):

$$\underline{\mathsf{H}}_F = \left(\underline{\mathsf{N}}_1{}^\top \underline{\Sigma} + \underline{\mathsf{N}}_2{}^\top \underline{\Gamma}\right)^\top \left(\underline{\mathsf{G}}_1 + \underline{\mathsf{G}}_2\right)^{-1} \times \left(\underline{\mathsf{N}}_1{}^\top \underline{\Sigma} + \underline{\mathsf{N}}_2{}^\top \underline{\Gamma}\right) \quad . \tag{4}$$

In the case of a PS converted wave, the downgoing ray (index 1) corresponds to the P wave and the upgoing ray (index 2) is an S wave. Therefore matrices $\underline{\mathsf{N}}_1$ and $\underline{\mathsf{G}}_1$ are determined from traveltimes for the P wave, and $\underline{\mathsf{N}}_2$ and $\underline{\mathsf{G}}_2$ from S-traveltimes. The matrices $\underline{\Sigma}$ and $\underline{\Gamma}$ are configuration matrices associated with the trace coordinates, e.g., for a common-shot configuration $\underline{\Sigma} = \underline{0}$ (zero) and $\underline{\Gamma} = \underline{1}$ (the unit matrix) (Schleicher et al., 1993).

That the diffraction traveltime can be constructed from two segments means that the diffraction time surface along which the summation stack is carried out can be obtained by interpolation using (3) from the coarse input traveltime grid onto the fine migration grid. We have, however, shown (Vanelle and Gajewski, 2001b), that a hyperbolic variant of equation (3) yields better results than the parabolic approximation (3) itself. It is obtained by expanding τ^2 into a Taylor series, that reads

$$\tau^2(\mathbf{s},\mathbf{g}) = (\tau_0 + \mathbf{q}_0\,\Delta\mathbf{g} - \mathbf{p}_0\,\Delta\mathbf{s})^2 - 2\,\tau_0\Delta\mathbf{s}^\top \underline{\mathsf{N}}\,\Delta\mathbf{g} + \tau_0\Delta\mathbf{g}^\top \underline{\mathsf{G}}\,\Delta\mathbf{g} - \tau_0\Delta\mathbf{s}^\top \underline{\mathsf{S}}\,\Delta\mathbf{s} \quad . \tag{5}$$

The vectors and matrices are the same as in equation (3) and can be determined from traveltimes in a similar fashion as for the parabolic variant (Vanelle and Gajewski, 2001b).

The geometrical spreading can also be written in terms of second-order traveltime derivatives as shown by Hubral et al. (1992). Their results were, however, derived for monotypic waves. For a converted wave we must include an additional factor to allow for the discontinuity of the spreading at the interface (Červený et al., 1977). This leads to

$$\mathcal{L} = \frac{1}{v_s}\,\sqrt{\cos\vartheta_s\cos\vartheta_g}\,\sqrt{\frac{\cos\vartheta_1}{\cos\vartheta_2}}\,\frac{1}{\sqrt{|\mathrm{det}\underline{\mathsf{N}}|}}\,e^{-i\,\frac{\pi}{2}\kappa} \quad . \tag{6}$$

where the matrix $\underline{\mathsf{N}}$ is given by

$$\underline{\mathsf{N}} = \underline{\mathsf{N}}_1\,(\underline{\mathsf{G}}_1 + \underline{\mathsf{G}}_2)^{-1}\,\underline{\mathsf{N}}_2{}^\top \quad . \tag{7}$$

The angles ϑ_s and ϑ_g in (6) are the emergence and incidence angles at the source and receiver. ϑ_1 is the incidence angle at the interface and ϑ_2 the reflection angle. v_s is the velocity of the downgoing wave at the source. In the case of a PS converted wave this will be the P-velocity. Equation (6) is also valid for monotypic waves, where $\vartheta_1 = \vartheta_2$, and thus the discontinuity factor $\sqrt{\cos\vartheta_1/\cos\vartheta_2}$ equals 1. All angles can be computed from the slownesses, and therefore from traveltimes, e.g., for ϑ_s:

$$\cos\vartheta_s = \sqrt{1 - v_s^2\;\mathbf{p}_{0_1}\!\cdot\mathbf{p}_{0_1}} \quad . \tag{8}$$

The expression for the weight function resulting from (6) is

$$W(\xi_1,\xi_2,M) = \frac{1}{v_s}\,\sqrt{\cos\vartheta_s\cos\vartheta_g}\,\sqrt{\frac{\cos\vartheta_1}{\cos\vartheta_2}}\,\frac{|\mathrm{det}(\underline{\mathsf{N}}_1{}^\top\underline{\mathbf{\Sigma}} + \underline{\mathsf{N}}_2{}^\top\underline{\mathbf{\Gamma}})|}{\sqrt{|\mathrm{det}\underline{\mathsf{N}}_1\,\mathrm{det}\underline{\mathsf{N}}_2{}^\top|}}\,e^{-i\,\frac{\pi}{2}(\kappa_1+\kappa_2)} \quad , \tag{9}$$

where κ_i are the KMAH indices of the down- and upgoing ray branches. These need not be computed, if a suitable traveltime generation tool is used, as, e.g., the technique by Coman and Gajewski (2001), which outputs multi-valued traveltimes sorted for the KMAH index. Equation (9) for monotypic waves results in the equations given by Schleicher et al. (1993).

APPLICATION

A 2-D example is given: ray synthetic seismograms (Figure 1) were obtained for a two-layers model with a horizontal interface in a common-shot configuration. The P-velocity is α_1=5km/s

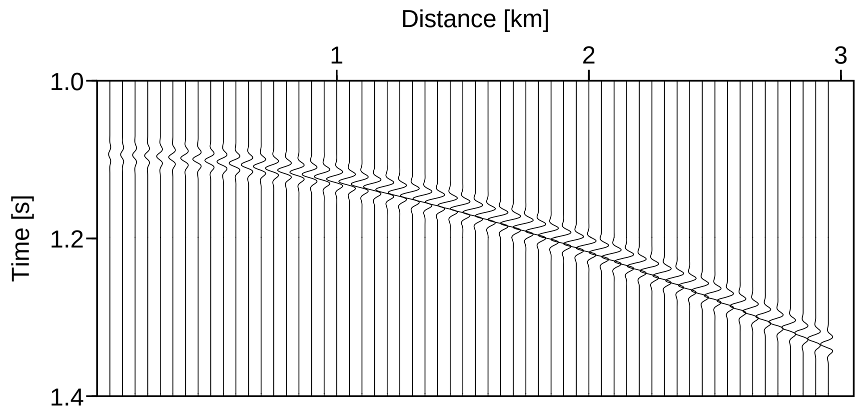

Figure 1: Synthetic common-shot section: The receiver spacing is 10m but only every fifth trace is shown here.

in the upper layer and α_2=6km/s in the lower layer that lies at a depth of 2km below the source. The S-velocity is $\beta_i = \alpha_i/\sqrt{3}$ and the density is given by $\rho = 1.7 + 0.2\alpha$ (ρ in g/cm^3 and α in km/s). 300 receivers with a spacing of 10m were distributed starting 10m away from a line source. Only PS reflections were considered in this example. Note that in two dimensions the analytic trace U must be $(i\omega)^{1/2}$-filtered. This corresponds to the time derivation in equation (1) in three dimensions.

Traveltimes were computed analytically on a 10m grid. They were the only input data for the determination of the migration weights in a 2-D variant of equation (2) as well as for the (hyperbolic) traveltime interpolation. All coefficients were computed from the hyperbolic approximation (5).

The migrated depth section is shown in Figure 2. Reflection coefficients were picked from the section and are compared to analytical results. Figure 3 shows good accordance between the two curves. The (negative) peak at a distance of 1.8km is a boundary effect caused by the limited extent of the receiver line, which provides sufficient illumination of the reflector only for distances smaller than 1.8km. This also causes the diffraction that shows in the migrated section (Figure 2).

CONCLUSIONS

We have presented a new strategy for true-amplitude migration of PS converted waves. It is based on the determination of all required properties alone from traveltime data. The traveltime tables need only be sampled on coarse grids, leading to considerable savings in storage. The *on-the-fly* computation of migration weight functions as well as a fast and highly accurate traveltime interpolation scheme reduce the necessary amount of computational time. A numerical example shows good accordance between the reconstructed reflector and theoretical values in terms of position as well as in reflection coefficients.

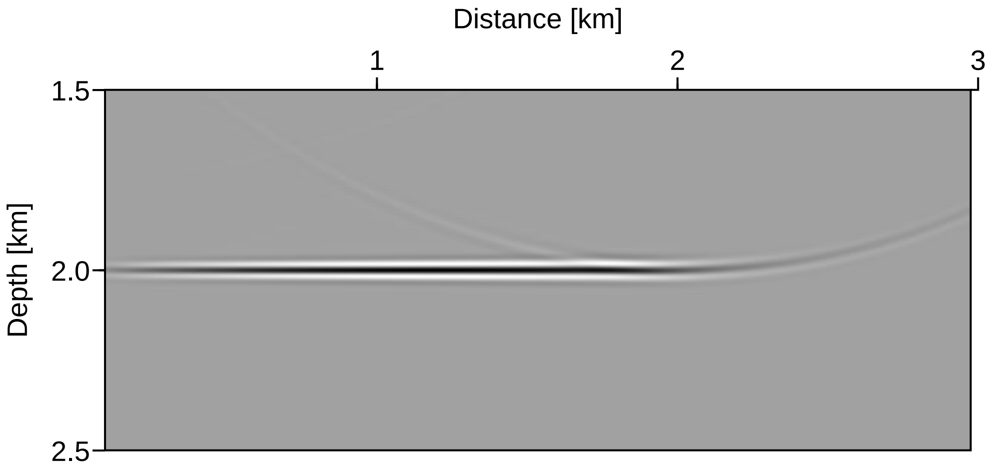

Figure 2: Migrated depth section: The reflector was migrated to the correct position. The amplitude matches the reflection coefficient. The diffraction is a boundary effect (see text).

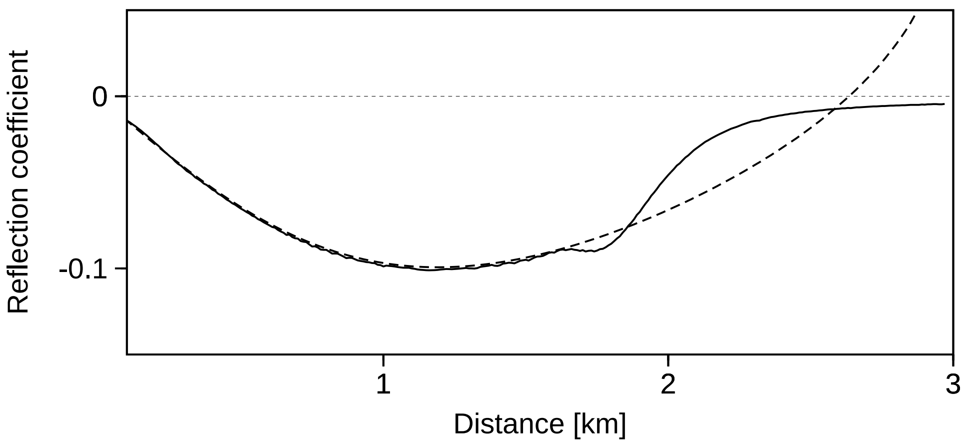

Figure 3: Recovered reflection coefficients from the common-shot gather in Figure 1: Solid line: picked reflection coefficients from the migrated section in figure 2. The peak at 1.8km is a boundary effect (see text). Dashed line: analytical values for the reflection coefficients.

PUBLICATIONS

The method of traveltime-based true-amplitude migration was introduced by Vanelle and Gajewski (2001c). More details on the traveltime interpolation and the determination of the coefficients can be found in Vanelle and Gajewski (2001c). In Vanelle and Gajewski (2001a) the authors describe how the optimum migration aperture can be obtained from traveltimes.

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