Attenuation and dispersion of seismic waves in 3-D randomly inhomogeneous, porous rocks

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ABSTRACT

A major cause of intrinsic wave attenuation in porous fluid-saturated structures with micro- and mesoscopic heterogeneities is the mechanism of wave-induced fluid flow. Whereas in stratified media this effect has been studied in detail for random and periodic arrangements of the heterogeneities, in 3-D space the models of frequency-dependent attenuation and dispersion are based on the assumption of spatial periodicity. We develop a model for 3-D randomly inhomogeneous poroelastic media. Within the precision of the Bourret approximation we study the dynamic characteristics of the coherent wavefield. Attenuation and dispersion depend on linear combinations of the spatial correlations of the fluctuating poroelastic parameters. The observed frequency dependence is typical for a relaxation phenomenon. In particular, we find that the low-frequency asymptote of the attenuation coefficient of a plane P-wave is proportional to the square of frequency. At high frequencies the attenuation coefficient becomes proportional to the square root of frequency.

INTRODUCTION

Understanding seismic wave propagation in porous fluid-saturated rocks is of great importance for the detection and exploitation of oil and gas reservoirs. In particular, attenuation and dispersion are wavefield characteristics which contain valuable information about the structure and composition of the reservoir. One major cause of seismic attenuation is due to the mechanism of wave-induced fluid flow. That means a passing wave creates local pressure gradients within the fluid phase and the resulting fluid flow is accompanied with internal friction until the pore pressure is equilibrated again. In fact, this attenuation mechanism is a dissipation process, where seismic wave energy is converted into heat. The fluid flow can take place on various length scales: for example from compliant fractures into the background porespace (so-called squirt flow), or between mesoscopic heterogeneities like fluid patches in partially saturated rocks.

There exist several models which predict frequency-dependent attenuation and dispersion due to this induced fluid flow. In layered structures the fluid flow degenerates to an 'inter-layer' flow from more compressible into stiffer layers during a compression cycle of the wave (and vice versa during extension). For periodic and random stratifications Gurevich and Lopatnikov (1995) and Gelinsky et al. (1998) developed models using the apparatus of statistical wave propagation. According to these studies, the low frequency asymptote of seismic attenuation (expressed through the reciprocal quality factor Q^{-1}) is proportional to frequency ω for periodic and $\sqrt{\omega}$ for random layering. In the high-frequency regime both types of layering yield $Q^{-1} \propto \omega^{-1/2}$. Recently, Pride and Berryman (2003) and Pride et al. (2003) developed a model of attenuation and dispersion due to squirt and mesoscopic flow in the framework of double porosity media. Based on energy considerations Johnson (2001) proposed a model for patchy saturation, which practically yields the same attenuation behavior than that predicted be Pride et al. (2003) for mesoscopic fluid patches. The theories of Pride et al. and Johnson make implicitly use of the assumption of spatial periodicity and result for 1-D and 3-D structures in the same low-frequency asymptote $Q^{-1} \propto \omega$. There is currently no model of seismic attenuation and dispersion for 3-D randomly heterogeneous structures.

We develop a model of seismic attenuation and dispersion for 3-D randomly inhomogeneous poroelastic media using the theory of statistical wave propagation. In particular, we study signatures of the coherent wavefield in the so-called Bourret approximation. This perturbation approach has been applied in acoustic scattering (Rytov et al., 1989) and elastic scattering (Gold et al., 2000). For randomly layered poroelastic media Gurevich and Lopantikov (1995) also employed the Bourret approximation in order to compute an effective, complex P-wavenumber, which describes conversion scattering from fast P-waves into Biot's slow waves. At low frequencies the slow wave corresponds to the process of pore pressure relaxation. That means the mechanism of conversion scattering provides a possibility to study the attenuation due to wave-induced fluid flow in the framework of scattering theories. The present work can be understood as a generalization to 3-D space of the previously proposed dynamic models of Gurevich and Lopantikov (1995), Gelinsky et al. (1998), and Shapiro and Hubral (1999).

In the light of previous results for randomly layered porous media, it is useful to introduce some simplifications from the very beginning. We restrict our analysis to low frequencies and to weak-contrast media. That means a) the wavelength is much larger than the heterogeneities so that conventional scattering is small and b) frequency is much smaller than the critical Biot frequency which implies that the Biot global flow mechanism is neglected and the P-wavenumber k_p is real. At low frequencies the slow wave is much slower than the fast P-wave and therefore the ratio of k_p and slow P-wavenumber k_{ps} is a small parameter:

$$\frac{|k_p|}{|k_{ps}|} \ll 1 \,. \tag{1}$$

Wherever applicable we make use of relation (1), being aware of the underlying low-frequency assumption.

FORMULATION OF THE PROBLEM

Biot's equation of poroelasticity

In order to study dynamic effects of seismic waves in porous media, we base our analysis on Biot's equation of poroelasticity (Biot, 1962). Using index notation – summation over repeated indices is assumed and partial derivatives are denoted as $_{,i}$ or ∂_i – we can write the equations of motion in the frequency domain (the time-harmonic dependency $\exp(-i\omega t)$ is omitted)

$$\begin{aligned} \rho\omega^2 u_i + \rho_f \omega^2 w_i + \tau_{ij,j} &= 0 \\ \rho_f \omega^2 u_i + \rho^\star \omega^2 w_i - p_{,i} &= 0 \,, \end{aligned}$$

where $\mathbf{u}$ and $\mathbf{w}$ are the solid and relative fluid displacement vectors, respectively. The latter is defined as $w_i = \phi(U_i - u_i)$ with porosity ϕ and fluid displacement vector $\mathbf{U}$. τ_{ij} is the total stress tensor, p the fluid pressure. The material parameters are given by the bulk density ρ, the density of the fluid ρ_f and $\rho^\star = \frac{i}{\omega}\frac{\eta}{\kappa}$, where η is viscosity and κ permeability. If the density of the solid is ρ_g, the bulk density can be expressed as $\rho = \phi\rho_f + (1-\phi)\rho_g$. In order to obtain a system of two coupled wave equations in the displacement vectors $\mathbf{u}$ and $\mathbf{w}$, we complement the equations of motion with the stress-strain relations for an isotropic poroelastic medium

$$\begin{aligned} \tau_{ij} &= \mu[u_{i,j} + u_{j,i} - 2\delta_{ij} u_{j,j}] + \delta_{ij}[H u_{j,j} + \alpha M w_{j,j}] \\ p &= -\alpha M u_{j,j} - M w_{j,j} \,. \end{aligned}$$

Here μ is the shear modulus, $H = P_d + \alpha^2 M$ the saturated P-wave modulus, and P_d the P-wave modulus of the dry rock frame. The poroelastic moduli α and M are defined as $\alpha = 1 - K_d/K_g$ and $M = [(\alpha-\phi)/K_g + \phi/K_f]^{-1}$. The quantities K_g, K_d, K_f denote the bulk moduli of the solid part (grains), the dry rock frame, and the pore fluid, respectively.
It is expedient to write the above system of wave equations in matrix form:

$$\begin{bmatrix} L^1_{ik} & L^2_{ik} \\ L^3_{ik} & L^4_{ik} \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix} = \mathbf{0} \,, \tag{2}$$

where we defined the linear differential operators as follows

$$
\begin{aligned}
L^1_{ik} &= \rho\omega^2\delta_{ik} + \partial_j\mu[\delta_{jk}\partial_i + \delta_{ik}\partial_j - 2\delta_{ij}\partial_k] + \partial_i H\partial_k && (3)\\
L^2_{ik} &= \rho_f\omega^2\delta_{ik} + \partial_i\alpha M\partial_k && (4)\\
L^3_{ik} &= L^2_{ik} && (5)\\
L^4_{ik} &= \rho^\star\omega^2\delta_{ik} + \partial_i M\partial_k\,. && (6)
\end{aligned}
$$

In the following we pursue a Green's function approach in order to solve matrix equation (2). The point source response of the system can be formulated as follows (Pride and Haartsen, 1996):

$$
\begin{bmatrix} u_i \\ w_i \end{bmatrix} = \begin{bmatrix} G^F_{ik} & G^f_{ik} \\ G^f_{ik} & G^w_{ik} \end{bmatrix} \begin{bmatrix} F^0_k \\ f^0_k \end{bmatrix}, \tag{7}
$$

where F^0_k and f^0_k represent force densities of the form $\mathbf{F} = \mathbf{F}^0\delta(\mathbf{r}-\mathbf{r}')$ and $\mathbf{f} = \mathbf{f}^0\delta(\mathbf{r}-\mathbf{r}')$, respectively. Here, δ denotes the Dirac delta function. The point source response is described by three Green tensors G^F_{ik}, G^f_{ik}, and G^w_{ik}. Explicit expressions for the G_{ik} are listed in the appendix.

Poroelastic scattering series

In randomly inhomogeneous porous media, all poroelastic parameters can be presented as random fields $X(\mathbf{r})$. To be more specific, we assume that these random fields are the sum of a constant background value, $\bar{X}(\mathbf{r})$ and a fluctuating part, $\tilde{X}(\mathbf{r})$, so that $X = \bar{X} + \tilde{X}$. The average over the ensemble of the realizations (denoted by $\langle\cdot\rangle$) of $\tilde{X}$ is zero: $\langle\tilde{X}\rangle = 0$. Consequently, the differential operators L_{ik} can be also written as $L_{ik} = \bar{L}_{ik} + \tilde{L}_{ik}$, where the perturbing operator $\tilde{L}_{ik}$ satisfies $\langle\tilde{L}_{ik}\rangle = 0$ (see also Karal and Keller, 1964). This implies that matrix equation (2) modifies to

$$
\begin{bmatrix} \bar{L}^1_{ik} & \bar{L}^2_{ik} \\ \bar{L}^3_{ik} & \bar{L}^4_{ik} \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix} + \begin{bmatrix} \tilde{L}^1_{ik} & \tilde{L}^2_{ik} \\ \tilde{L}^3_{ik} & \tilde{L}^4_{ik} \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix} = \mathbf{0}\,. \tag{8}
$$

In the most general case, the perturbing operators contain fluctuations of all poroelastic moduli and densities. In analogy to the elastic case (Gubernatis et al., 1977) equation (8) can be converted in an integral equation of the form

$$
\begin{bmatrix} u_i \\ w_i \end{bmatrix} = \begin{bmatrix} u^0_i \\ w^0_i \end{bmatrix} + \int_V dV \begin{bmatrix} G^F_{ij} & G^f_{ij} \\ G^f_{ij} & G^w_{ij} \end{bmatrix} \begin{bmatrix} \tilde{L}^1_{jk} & \tilde{L}^2_{jk} \\ \tilde{L}^3_{jk} & \tilde{L}^4_{jk} \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix}, \tag{9}
$$

where the total wavefields $\mathbf{u}$ and $\mathbf{w}$ are composed of wavefields propagating in the homogeneous background medium ($\mathbf{u}^0$ and $\mathbf{w}^0$) and scattered wavefields (represented by the second term). The scattered wavefields vanish if there are no fluctuations in the medium parameters. Obviously, the integration volume encompasses the inhomogeneous part of the medium. Since equation (9) must be also true for the Green tensors G_{ij}, we can write

$$
\begin{bmatrix} G^F_{im} & G^f_{im} \\ G^f_{im} & G^w_{im} \end{bmatrix} = \begin{bmatrix} {}^0G^F_{im} & {}^0G^f_{im} \\ {}^0G^f_{im} & {}^0G^w_{im} \end{bmatrix} + \int_V dV \begin{bmatrix} {}^0G^F_{ij} & {}^0G^f_{ij} \\ {}^0G^f_{ij} & {}^0G^w_{ij} \end{bmatrix} \begin{bmatrix} \tilde{L}^1_{jk} & \tilde{L}^2_{jk} \\ \tilde{L}^3_{jk} & \tilde{L}^4_{jk} \end{bmatrix} \begin{bmatrix} G^F_{km} & G^f_{km} \\ G^f_{km} & G^w_{km} \end{bmatrix}. \tag{10}
$$

In order to simplify the equations that follow, we introduce a shorthand notation. The latter equation can be symbolically re-written as

$$
\mathbf{G} = \mathbf{G}^0 + \int \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}\,. \tag{11}
$$

We must, however, keep in mind that all these quantities represent matrices – whose elements are tensors of rank two – and matrix multiplication rules apply.

POROELASTIC BOURRET APPROXIMATION

Mean Green's tensor

Since the following calculations are formally very similar to the elastic case (Gold et al., 2000) we only provide the key equations necessary in subsequent sections. Expanding the scattering series (11) we obtain

$$\mathbf{G} = \mathbf{G}^0 + \int \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0 + \iint \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0 + \iiint \ldots\,. \tag{12}$$

Averaging this equation and regrouping the scattering terms yields a matrix containing the mean Green tensors

$$\bar{\mathbf{G}} = \mathbf{G}^0 + \iint \mathbf{G}^0\mathbf{Q}\bar{\mathbf{G}}\,, \tag{13}$$

where the matrix $\mathbf{Q}$ is defined as

$$\mathbf{Q} = \left\langle \tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}} + \int \tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}} + \iint \ldots \right\rangle\,, \tag{14}$$

and corresponds to the kernel-of-mass operator in the acoustic formulation (Rytov et al., 1989). Equation (13) is the poroelastic Dyson equation. If only the first term in the expansion of $\mathbf{Q}$ is retained, we obtain an equation of the mean Green tensor in the Bourret approximation. It is important to note that the elements of $\mathbf{Q}$ only contain terms involving the second statistical moment of the fluctuating parts of the $\tilde{L}_{ik}$'s. Higher-order correlations are neglected. Since equation (13) contains a double convolution, it is expedient to work with its spatial Fourier transform (see the appendix for notation):

$$\bar{\mathbf{g}} = \mathbf{g}^{\mathbf{0}} + (8\pi)^3\mathbf{g}^{\mathbf{0}}\mathbf{q}\,\bar{\mathbf{g}}\,. \tag{15}$$

Unlike in the acoustic case, this equation cannot explicitly solved for the mean Green tensors. In fact, we have a system of four tensorial equations for the three unknown mean Green tensors. Carrying out the necessary matrix multiplications we find that this system splits up into two pairs of coupled equations. Since we are only interested in the characteristics of the P-wave (see next section) we analyze only those two equations that involve $\bar{g}^F_{ik}$. We obtain

$$\bar{g}^F = g^F + (8\pi)^3[g^F q^1\bar{g}^F + g^F q^2\bar{g}^f + g^f q^3\bar{g}^F + g^f q^4\bar{g}^f] \tag{16}$$

$$\bar{g}^f = g^f + (8\pi)^3[g^f q^1\bar{g}^F + g^f q^2\bar{g}^f + g^w q^3\bar{g}^F + g^w q^4\bar{g}^f]\,, \tag{17}$$

where we omitted subscripts for brevity. The quantities g without upper bar denote the homogeneous space Green tensors.

Effective P-wave number

In order to extract an effective wavenumber from equations (16) and (17) we have to introduce some simplifications. Let us assume that the medium's fluctuations are weak or frequencies are low so that the fluctuations of the wavefield are small. Then, we can also assume that the mean Green tensor $\bar{g}^F_{ik}(\mathbf{K})$ is of the same functional form as $g^F_{ik}(\mathbf{K})$, however, involving some effective P-wave number (and also effective bulk density). Furthermore, solving equations (16) and (17) by a perturbation approach, we can use $\bar{g}^f \approx g^f$ as the lowest order approximation. Since we are not interested in the mean Green tensor itself, but only in the effective P-wave number, we construct a simple case, where most of the Green tensor components vanish. This can be achieved using the following procedure: We consider an incoming P-wave propagating in z-direction (i.e. only u_3 is nonzero). Then, only the tensor component $i = j = 3$ needs to be analyzed. Noting that in this case the Green tensor $g^F_{ik}(\mathbf{K})$ yields only a contribution for the spatial wavenumber $K = k_p$ (see equation (34)) we obtain from equation (16) after algebraic manipulations

$$\bar{k}_p \approx k_p\left(1 + \frac{4\pi^3}{\rho\omega^2}q^1_{33}\right)\,. \tag{18}$$

We neglected terms that contain combinations of the tensor components q^i_{33}. This introduces no additional inaccuracy because higher-order correlations are neglected within the precision of the Bourret approximation.
The remaining problem is the evaluation of q^1_{33}. In explicit form, we obtain from the first term in the expansion of $\mathbf{Q}$

$$\begin{aligned} Q^1_{ik}(\mathbf{r}'-\mathbf{r}'') &= \langle \tilde{L}^1_{ij}(\mathbf{r}')G^F_{jl}(\mathbf{r}'-\mathbf{r}'')\tilde{L}^1_{lk}(\mathbf{r}'') + 2\tilde{L}^1_{ij}(\mathbf{r}')G^f_{jl}(\mathbf{r}'-\mathbf{r}'')\tilde{L}^2_{lk}(\mathbf{r}'') + \\ & \quad \tilde{L}^2_{ij}(\mathbf{r}')G^w_{jl}(\mathbf{r}'-\mathbf{r}'')\tilde{L}^2_{lk}(\mathbf{r}'')\rangle \end{aligned} \tag{19}$$

It is interesting to note that in the elastic limit, only the first term of Q^1_{ik} is nonzero. That means poroelastic effects are also caused by the other two terms, which we will consider in detail.

Let us now specify the perturbing operators $\tilde{L}_{ik}$ for a particular situation. Since we are mainly interested in the poroelastic effect produced by conversion scattering into the slow wave (see Introduction), we assume that there are no fluctuations in the shear modulus. We also neglect fluctuations of the densities ρ, ρ_f, and $\rho^\star$. This is possible because of the restriction to low frequencies. Finally, we assume that the fluctuations in the parameter α are smaller than those of the parameters M and H and therefore can be neglected. The incurring error will be small if $K_d \ll K_g$. These simplifications yield

$$\tilde{L}^1_{ij} = \partial_i \tilde{H} \partial_j \qquad \tilde{L}^2_{ij} = \alpha \partial_i \tilde{M} \partial_j \,. \tag{20}$$

The operator $\tilde{L}^4_{ik}$ does not appear in equation (19) and is not further analyzed. The computation of the second and third term of Q^1_{33} is now straightforward. Following closely the computations in Gold et al. (2000) we obtain for the first term in the wavenumber domain

$$^{(1)}q^1_{33} = \frac{1}{8\pi^3}\frac{\alpha^2 M}{H P_d}\left(k_p^2 B_{HH}(0) + k_p k_{ps}^2 \int_0^\infty dr B_{HH}(r)\exp[ik_{ps}r]\sin(k_p r)\right) \tag{21}$$

for the second term

$$^{(2)}q^1_{33} = -\frac{1}{4\pi^3}\frac{\alpha^2}{P_d}\left(k_p^2 B_{MH}(0) + k_p k_{ps}^2 \int_0^\infty dr B_{MH}(r)\exp[ik_{ps}r]\sin(k_p r)\right) \tag{22}$$

and for the third term

$$^{(3)}q^1_{33} = \frac{1}{8\pi^3}\frac{\alpha^2}{N}\left(k_p^2 B_{MM}(0) + k_p k_{ps}^2 \int_0^\infty dr B_{MM}(r)\exp[ik_{ps}r]\sin(k_p r)\right) \,. \tag{23}$$

Here, B_{HH}, B_{MH} and B_{MM} denote the (cross-) correlation functions of the random fields $\tilde{M}$ and $\tilde{H}$. To be consistent with our low frequency assumption, we replace the $\sin$ function by its argument. Substituting these expressions into equation (18) we obtain the final result for the effective P-wavenumber

$$\bar{k}_p = k_p\left[1 + \sum_{n=1}^{3} c_n\left(B_n(0) + k_{ps}^2 \int_0^\infty dr\, r\, B_n(r)\exp[ik_{ps}r]\right)\right] \,, \tag{24}$$

with the coefficients

$$c_1 = \frac{1}{2}\frac{\alpha^2 M}{P_d} \qquad c_2 = -2c_1 \qquad c_3 = c_1 \tag{25}$$

and $B_1 = B_{HH}$, $B_2 = B_{MH}$ and $B_3 = B_{MM}$ denote the normalized (cross-) correlation functions. It is important to note that the above expression describes only the process of conversion scattering from fast to slow P-waves. The contribution of purely elastic scattering is taken out (the corresponding result would include an additional term invloving the correlation function B_{HH}). Equation (24) is the central result of this paper. In subsequent sections we will analyze the properties of k_p. The structure of equation (24) is similar to the 1-D result (see equation 56 in Gurevich and Lopatnikov, 1995).

ATTENUATION AND DISPERSION

>From the complex P-wavenumber (24) we can extract the attenuation and dispersion characteristics. By definition, the real part of $\bar{k}_p$ yields the phase velocity $c(\omega) = \Re\{\bar{k}_p\}$ and the imaginary part yields the attenuation coefficient α. Instead of α, we use for the characterization of attenuation the dimensionless, reciprocal quality factor Q^{-1}, which for low-loss media is given by $Q^{-1} = 2\alpha/\Re\{\bar{k}_p\}$.

One goal of our study is to infer the low- and high-frequency asymptotes of attenuation due to wave-induced fluid flow. At low frequencies we can approximate the exponential in equation (24) by 1. Then we immediately have $\alpha \propto \omega^2$ or in terms of the quality factor

$$Q^{-1} \propto \omega \,. \tag{26}$$

Obviously, this asymptote only exists if $\int_0^\infty dr\, rB(r)$ has a finite value. This is, however, the case for a large class of correlation functions.
At high frequencies only the behavior of $B(r)$ at small arguments is important. Assuming that the correlation function can be expanded around the origin in the form $B(r) = 1 - r + O(r^2)$, we can evaluate the integral in equation (24) and obtain

$$Q^{-1} \propto \frac{1}{\sqrt{\omega}} \,. \tag{27}$$

The same asymptote has been found in 1-D/3-D periodic and 1-D random structures.

Let us consider an example. For a typical porous sandstone with porosity $\phi = 17\%$ and permeability $\kappa = 250$mD we use the following background parameters: $P_d = 29$GPa, $\alpha = 0.67$, and $M = 11$GPa. The pore fluid is water with viscosity $\eta = 0.001$Pas. The correlation function is of the form $B_n(r) = \sigma_n^2 \exp[-|r/a|]$, where σ_n^2 is the variance of the fluctuations and a the correlation length. Assuming that we have only fluctuations in the poroelastic parameter M with $B_{MM}(0) = \sigma_{MM}^2 = 0.08$ and a correlation length $a = 10$cm we obtain the frequency dependence of Q^{-1} and c as shown in Figures (1) and (2). The frequency is normalized by Biot's critical frequency, which in this example is $f_c \approx 112$kHz. Both curves are typical for a relaxation phenomenon. The low and high frequency asymptotic behavior of attenuation is clearly in agreement with equations (26) and (27). The low frequency limit of phase velocity is determined by the first two terms in equation (24) and results in a frequency-independent velocity shift. At high frequencies the phase velocity is given by the speed of the background medium (that is $c = 3000$m/s in our example).

DISCUSSION AND CONCLUSIONS

Our analysis is not yet complete with respect to the following items:
1) The validity range of the used approximation has to be determined properly. The most important assumptions made are the restriction to low frequencies and to weak-contrast media. We note that these restrictions are consistent with the known range of validity for the acoustic Bourret approximation (Rytov et al., 1989): $\sigma_n^2(ka)^2 \ll 1$, where σ_n^2 is the variance of fluctuations. We expect that a similar condition can be found in the poroelastic case.
2) In our analysis we took into account only the fluctuations of the poroelastic parameters M and H. We think that the proposed model can be extended in an analogous way if additional fluctuating parameters are introduced. This is of particular interest when studying the relationship between rock-heterogeneity and partial saturation, where the cross-correlations between rock and fluid properties become important (Shapiro and Müller, 1999).
3) In order to construct a quantitative model for the case of partial rock saturation, the present approach has to be modified in such a way that the low- and high frequency limits are connected with the known (exact) bounds of Gassmann-Wood and Gassmann-Hill, respectively (for the 1-D case see Müller and Gurevich, 2003).

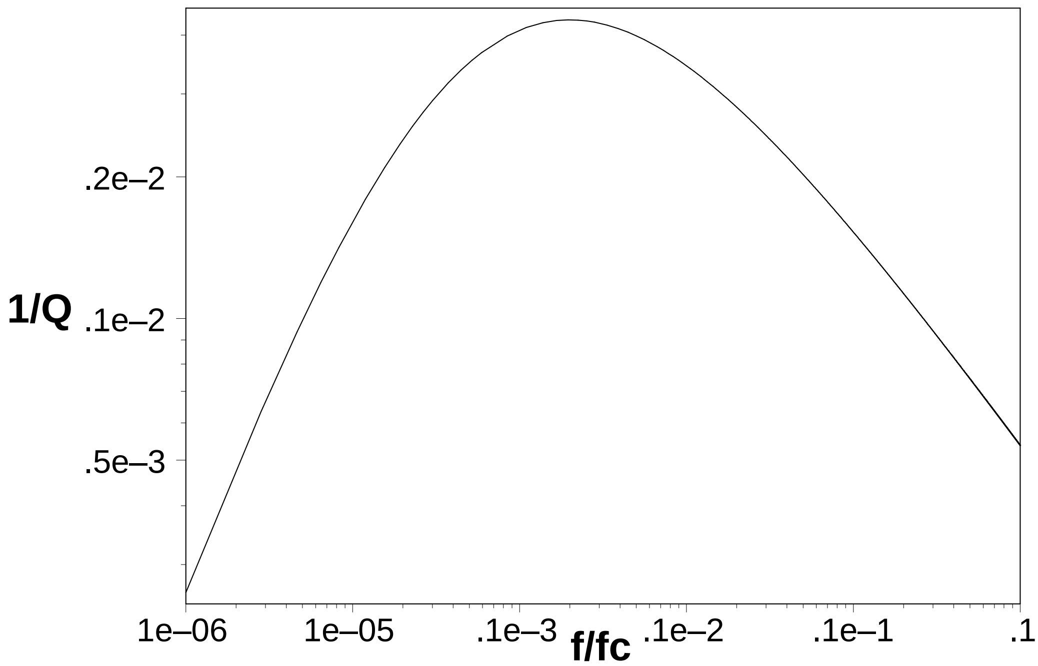

Figure 1: Reciprocal quality factor versus frequency (normalized by Biot's critical frequency f_c). The medium parameters are those of a porous, water-saturated sandstone (see text).

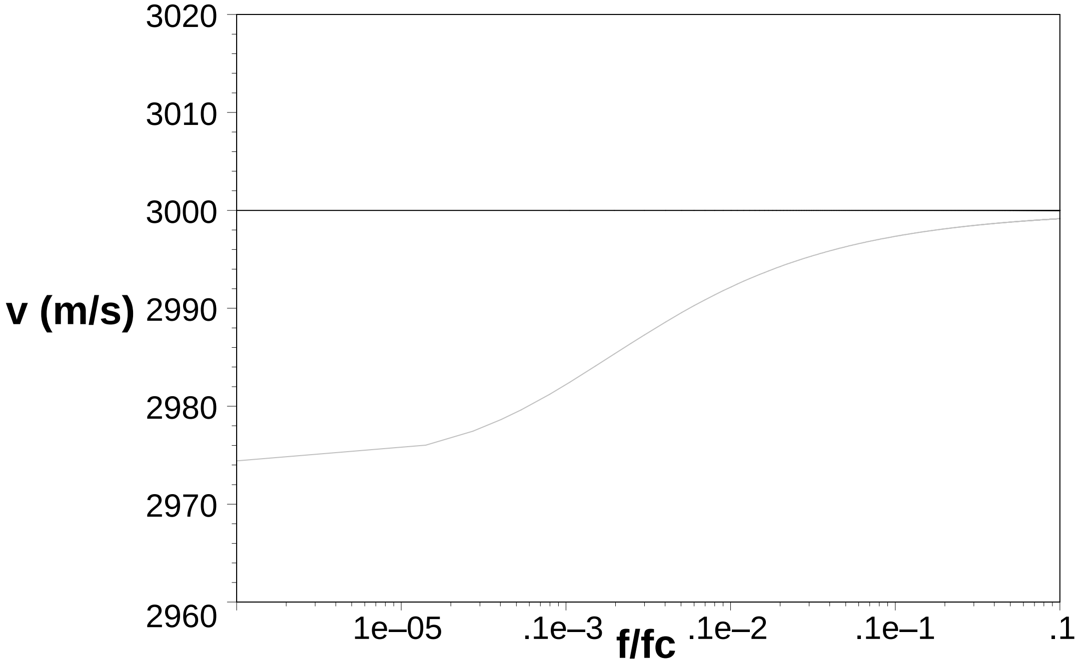

Figure 2: P-wave phase velocity versus frequency (black curve). P-wave velocity in the background medium ($\sigma_{MM}^2 = 0$) is shown as gray line. The medium parameters are the same as in Figure (1).

We proposed a model for frequency dependent attenuation and dispersion in 3-D randomly inhomogeneous porous rocks accounting for the effect of wave-induced fluid flow. In our approach the dynamic characteristics depend on the correlation properties of the medium fluctuations. Explicit results for $Q^{-1}(\omega)$ and $c(\omega)$ can be obtained for certain correlation functions. The form of attenuation and dispersion curves are typical for a relaxation mechanism. The low-frequency behavior of attenuation is found to be $Q^{-1} \propto \omega$, whereas at high frequencies $Q^{-1} \propto \omega^{-1/2}$. It is very interesting to note that these asymptotes coincide with those predicted by the periodicity-based approaches (see Introduction). Consequently, in 3-D space the observed frequency dependency of attenuation due to fluid flow has universal character independent of the type of disorder (periodic or random). This result is somewhat unexpected if we remember that in 1-D space the attenuation asymptotes are different for periodic and random structures.

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APPENDIX A

The complete set of Green tensors for a homogeneous and isotropic poroelastic continuum – including electro-seismic coupling – was derived by Pride and Haartsen (1996). We list only those parts of the Green tensors, which are related to poroelastic wave propagation. Furthermore, we can simplify these tensors if we use $|k_p|/|k_{ps}| \ll 1$. We obtain:

$$G^F_{ij}(\mathbf{r},\mathbf{r}_0) = \frac{1}{4\pi\rho\omega^2}\left(\left[k_s^2\delta_{ij}+\partial_i\partial_j\right]\frac{e^{ik_sR}}{R}-\partial_i\partial_j\frac{e^{ik_pR}}{R}\right)-\frac{\alpha^2M^2}{H^2}\frac{1}{4\pi\rho^\star\omega^2}\partial_i\partial_j\frac{e^{ik_{ps}R}}{R} \quad (28)$$

$$G^f_{ij}(\mathbf{r},\mathbf{r}_0) = \frac{\alpha M}{H}\frac{1}{4\pi\rho^\star\omega^2}\partial_i\partial_j\frac{e^{ik_{ps}R}}{R} \quad (29)$$

$$G^w_{ij}(\mathbf{r},\mathbf{r}_0) = -\frac{1}{4\pi\rho^\star\omega^2}\partial_i\partial_j\frac{e^{ik_{ps}R}}{R}\,, \quad (30)$$

where $R = |\mathbf{r}-\mathbf{r}_0|$. In fact, in homogeneous and isotropic media the Green tensors only depend on R. The wavenumbers are defined as

$$k_p = \omega\sqrt{\frac{\rho}{H}} \qquad k_s = \omega\sqrt{\frac{\rho}{\mu}} \qquad k_{ps} = \sqrt{\frac{i\omega\eta}{\kappa N}}\,. \quad (31)$$

Note that the first term of G^F_{ij} is identical with the elastodynamic Green tensor (Gubernatis et al., 1977). We define the spatial Fourier transform pair in the following way:

$$G_{ij}(\mathbf{r}-\mathbf{r}') = \int d^3K\, g_{ij}(\mathbf{K})\exp[i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')] \quad (32)$$

$$g_{ij}(\mathbf{K}) = \frac{1}{(2\pi)^3}\int d^3R\, G_{ij}(\mathbf{r}-\mathbf{r}')\exp[-i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')]\,. \quad (33)$$

In the wavenumber domain the above Green tensors read

$$g^F_{ij}(\mathbf{K}) = -\frac{1}{8\pi^3}\frac{1}{\rho\omega^2}\left(\frac{k_s^2\delta_{ij}-K_iK_j}{k_s^2-K^2}+\frac{K_iK_j}{k_p^2-K^2}\right)-\frac{1}{8\pi^3}\frac{\alpha^2M^2}{H^2}\frac{1}{\rho^\star\omega^2}\frac{K_iK_j}{k_{ps}^2-K^2} \quad (34)$$

$$g^f_{ij}(\mathbf{K}) = \frac{1}{8\pi^3}\frac{\alpha M}{H}\frac{1}{\rho^\star\omega^2}\frac{K_iK_j}{k_{ps}^2-K^2} \quad (35)$$

$$g^w_{ij}(\mathbf{K}) = -\frac{1}{8\pi^3}\frac{1}{\rho^\star\omega^2}\frac{K_iK_j}{k_{ps}^2-K^2}\,. \quad (36)$$