3-D CRS TRAVELTIME HYPERBOLIC APPROXIMATIONS FOR REFLECTIONS AND DIFFRACTION EVENTS

C. Lima, G. Garabito, and P. Chira

email: *limacca@ufpa.br*

keywords: *CRS traveltime approximation, Seismic Modeling, 3-D hipothetical wavefronts*

ABSTRACT

The Zero-Offset (ZO) Common-Reflection-Surface (CRS) stack is a macro-model independent seismic reflection imaging method that simulates a ZO volume or section from multi-coverage reflection pre-stack data. This method has been established as an improvement and alternative of the conventional Normal-Moveout/Dip-Moveout (NMO/DMO) processing. Over the past years it has been successfully applied both to 2-D and 3-D synthetic and real seismic data. It provides important wavefield attributes or parameters for several applications, e.g. migration, inversion and interpretation. It uses as operator a second-order hyperbolic traveltime approximation in the vicinity of a central ray. In 3-D, for a normal or ZO central ray, this operator depends on eight parameters that are determined by means of coherence analyzes procedures. In this work, we examine the 3-D ZO CRS operator for reflection and diffraction events with its respective true traveltimes. The results of these comparisons demonstrate that the 3-D ZO CRS operator has a good fit with the true traveltime surface.

INTRODUCTION

In the last years a lot of macro-model independent imaging methods have appeared. The CRS Stack technique is in this group. The section or volume simulated by the CRS presents better signal-to-noise ratio and resolution then conventional methods, e.g NMO/DMO. The conventional methods are based on suppositions of 1-D velocity model and it use traveltime approximations only applicable on CMP (common-midpoint) data. The only one parameter that this conventional approximation, called NMO velocity stack, have a limited use in others attributes extraction of the seismic media or for depth velocity model inversion.The CRS method uses all multi-coverage seismic data and provides, additionally, important parameters from these data. It can be performed without any macro velocity model estimation, It is only necessary the velocity close to surface.

The CRS method have been demonstrate be more efficient then conventional methods when applied on synthetic and real data sets (e.g. Cristini et al. (2001); Bergler et al. (2002)). The ZO-CRS operator is based on a second order traveltime hyperbolic approximation in the vicinity of a normal central ray that depends on eight parameters in 3-D case. This stack parameters, obtained to make a CRS volume, describe the normal ray direction and wavefront curvatures of the normal (N) and normal incident point (NIP) wavefronts Hubral (1983). This parameters, that are also called wavefront attributes, have important applications, e.g. migration, inversion and interpretation. It can be determined by automatic search process with coherence analysis of the pre-stack seismic data. The first results of the 3-D CRS stack applied in synthetic data were presented by Cristini et al. (2001). In real data sets, the first results were presented by e.g. Cristini et al. (2002); and Bergler et al. (2002).

The CRS method is also efficient when applied on strong lateral velocity variations media data, structural complexity, low signal/noise ratio and poor data coverage. For a 3-D data set with very reduced azimuth, the number of parameters reduce to four Chira (2003), six to marine case Cardone et al. (2003).

In Chira-Oliva et al. (2003) we find the formalism and application examples of the 3-D ZO-CRS stack

operator for reflection events, and the operator for diffraction events, that depends on five parameters.
In this work we present the first numeric results about 3-D ZO-CRS traveltime surfaces associated with a reflected and diffracted central ray. This approximated 3-D ZO-CRS surfaces are compared with respective true traveltimes, calculated by ray tracing. This results are important for development of strategies to provide the eight 3-D CRS parameters and the consequent implementation of 3-D CRS stack to make ZO sections or volumes simulations.

THEORETICAL ASPECTS

Wavefront curvatures

In this work we used a 3-D model constituted by isovelocities layers separated for curved interfaces (Figure 1). The wavefront curvatures in any point of the same can be expressed analytically in term of the seismic parameters along the ray that connects the observation point with the origin of the ray (Hubral and Krey, 1980). Three laws requested for the calculation of the curvatures of the wave fronts along an arbitrary ray are presented. These curvature laws can be used to supply approximated direct solutions for the behavior of the amplitude of the body and head waves by ray method (Cerveny and Ravindra, 1971). The wavefront in any point of the same can be approximate for a surface in movement represented by

$$z = \frac{-1}{2}\mathbf{X}\,\hat{\mathbf{A}}\,\mathbf{X}^T, \tag{1}$$

where

$$\mathbf{X} = (x, y)\ , \quad \hat{\mathbf{A}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}, \tag{2}$$

being $\hat{A}$ the curvature matrix of the wavefront that is symmetric. Particularly, $\hat{A}_I$, $\hat{A}_T$ and $\hat{A}_R$, refer to the curvature matrix of reflected, refracted and incidents in a interface point wavefronts related to the system in movement in this point. Previously the (x_F, x_F, z_F) system is defined as an auxiliary system in all refraction and reflection points in the interface. Each interface can be approximate in a point of intersection of the ray in relation to the auxiliary system for the following second-order polynomial:

$$z_F = \frac{-1}{2}\mathbf{X}_F\,\hat{\mathbf{B}}\,\mathbf{X}_F^T, \tag{3}$$

where

$$\mathbf{X}_F = (x_F, y_F)\ , \quad \hat{\mathbf{B}} = \begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{pmatrix}, \tag{4}$$

$\hat{\mathbf{B}}$ is the interface curvature matrix and it is also symmetrical. The inverse matrix $\hat{\mathbf{R}}_A = \hat{\mathbf{A}}^{-1}$ are the curvature radii matrix.

Curvature propagation law

To calculate the curvature in P_1 we should meet the curvature matrix $\mathbf{X}_0$ in P_0 and the distance of P_0 to P_1. The distance is given by $v\ \Delta t$, where v is the velocity and Δt the time that the wavefront travels from P_0 to P_1. $\mathbf{I}$ is the 2×2 unitary matrix. Therefore,

$$\hat{\mathbf{R}}_{P_1} = \hat{\mathbf{R}}_{P_0} + v\ \Delta t\ \mathbf{I}, \tag{5}$$

the matrix $\hat{\mathbf{R}}_{P_1}(i = 0, 1)$ denotes the radii matrix in the point P_i and it is given by the inverse of $\hat{\mathbf{A}}_i$ matrix.

Refraction law

In a reflection point in an interface, the $\hat{\mathbf{A}}_I$, $\hat{\mathbf{A}}_R$ and $\hat{\mathbf{B}}$ matrices are defined in relation to the coordinate systems (x_I, y_I, z_I), (x_R, y_R, z_R) e (x_F, y_F, z_F). These three curvature matrices are related by:

$$\hat{\mathbf{A}}_R = \mathbf{D}^{-1}\,\mathbf{I}_R\,\left(\frac{v_R}{v_I}\,\mathbf{S}'\,\hat{\mathbf{A}}_I\,\mathbf{S}' + \rho\,\mathbf{S}_R^{-1}\,\hat{\mathbf{A}}_I\,\mathbf{S}_R^{-1}\right)\,\mathbf{I}_R\,\mathbf{D}, \tag{6}$$

where

$$\mathbf{I}_R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{S}' = \begin{pmatrix} -\cos\varepsilon_I/\cos\varepsilon_R & 0 \\ 0 & 1 \end{pmatrix},$$
$$\mathbf{S}_R = \begin{pmatrix} -\cos\varepsilon_R & 0 \\ 0 & 1 \end{pmatrix}, \rho' = \frac{v_R}{v_I}\ \cos\varepsilon_I + \cos\varepsilon_R\ . \qquad (7)$$

3-D TRAVELTIME HYPERBOLIC APPROXIMATIONS

Reflection events

The ZO 3-D CRS operator is given by

$$t_{ref}^2 = \left(t_0 + \frac{2}{v_0}\mathbf{w}_z \cdot \mathbf{m}\right)^2 + \frac{2\,t_0}{v_0}\left(\mathbf{m}^T \cdot\ \mathbf{T}\,\hat{\mathbf{N}}\,\mathbf{T}\,\mathbf{m} + \mathbf{h}^T \cdot\ \mathbf{T}\,\hat{\mathbf{M}}\,\mathbf{T}\,\mathbf{h}\right)\ , \qquad (8)$$

where

$$\mathbf{w}_z = \begin{pmatrix} \cos\varphi_0 & \sin\varphi_1 \\ \sin\varphi_0 & \cos\varphi_1 \end{pmatrix}, \hat{\mathbf{M}} = \begin{pmatrix} m_{00} & m_{01} \\ m_{01} & m_{11} \end{pmatrix}, \hat{\mathbf{N}} = \begin{pmatrix} n_{00} & n_{01} \\ n_{01} & n_{11} \end{pmatrix}, \qquad (9)$$

and the 2-D transformation matrix, **T**, is given by (Jäger, 1999)

$$\begin{aligned} \mathbf{T} &= \mathbf{D}_{zy} = \mathbf{D}_z(\varphi_0)\ \mathbf{D}_y(\varphi_1) \\ &= \begin{pmatrix} \cos\varphi_0 & -\sin\varphi_0 \\ \sin\varphi_0 & \cos\varphi_0 \end{pmatrix}\begin{pmatrix} \cos\varphi_1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned} \qquad (10)$$

or it can also be expressed by Höcht (2002)

$$\begin{aligned} \mathbf{T} &= \mathbf{D}_{zyz} = \mathbf{D}_z(\varphi_0)\ \mathbf{D}_y(\varphi_1)\ \mathbf{D}_z(\varphi_F) \\ &= \begin{pmatrix} \cos\varphi_0 & -\sin\varphi_0 \\ \sin\varphi_0 & \cos\varphi_0 \end{pmatrix}\begin{pmatrix} \cos\varphi_1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\varphi_F & -\sin\varphi_F \\ \sin\varphi_F & \cos\varphi_F \end{pmatrix} \end{aligned} \qquad (11)$$

where

$$\begin{pmatrix} \cos\varphi_F \\ \sin\varphi_F \end{pmatrix} = \frac{\mathbf{D}_y^T(\varphi_1)\ \mathbf{D}_z^T(\varphi_0)\ \mathbf{s}_F}{\sqrt{1-(\mathbf{w}_z\cdot\mathbf{s}_F)^2}}, \quad \mathbf{s}_F = \begin{pmatrix} \cos\beta_F \\ \sin\beta_F \end{pmatrix}, (-\phi < \beta_F < \phi) \qquad (12)$$

being t the traveltime along the stack surface in the 5-D data volume supplied by the same time, the two vector components mid-point **m** and the two vector components half-offset **h**. v_0 is the velocity in the vicinity of the normal ray emergency point in the surface. $\mathbf{W}_z$ is a two components vector that defines the normal ray direction in the measurement surface. φ_0 and φ_1 indicate the azimuth and the polar angle of the normal ray direction. The 2x2 symmetrical matrices, **M** and **N**, are the NIP(figure 1)and normal matrices of the wavefront curvatures in the surface. **T** is the transformation matrix that depends on $\mathbf{W}_z$ components. $\mathbf{S}_f$ is the unitary vector of a reference plan on the plane measurement surface. β_F is the azimuth of the unitary vector $\mathbf{S}_f$. Höcht (2002) defines the reference plan as a plan formed by a reference unitary vector $\mathbf{S}_f$ and the direction vector of the normal reflection ray. The T superscription indicates transpose matrix. Chira-Oliva et al. (2003) present the operator (7) in a simpler way, considering $\mathbf{A} = \mathbf{T}\hat{\mathbf{N}}\mathbf{T}$ and $\mathbf{B} = \mathbf{T}\hat{\mathbf{M}}\mathbf{T}$, for the appropriate search of the eight parameters.

Diffraction events

In this seismic events, we considered $\hat{\mathbf{M}} = \hat{\mathbf{N}}$ Chira-Oliva et al. (2003),

$$t_{dif}^2 = \left(t_0 + \frac{2}{v_0}\mathbf{w}_z \cdot \mathbf{m}\right)^2 + \frac{2\,t_0}{v_0}\left(\mathbf{m}^T \cdot\ \mathbf{T}\,\hat{\mathbf{M}}\,\mathbf{T}\,\mathbf{m} + \mathbf{h}^T \cdot\ \mathbf{T}\,\hat{\mathbf{M}}\,\mathbf{T}\,\mathbf{h}\right)\ , \qquad (13)$$

where the 3-D Common Diffraction Surface operator (CDS) depends on five parameters: two elements of the $\mathbf{w}_z$ vector and three elements of the $\hat{\mathbf{M}}$ matrix.

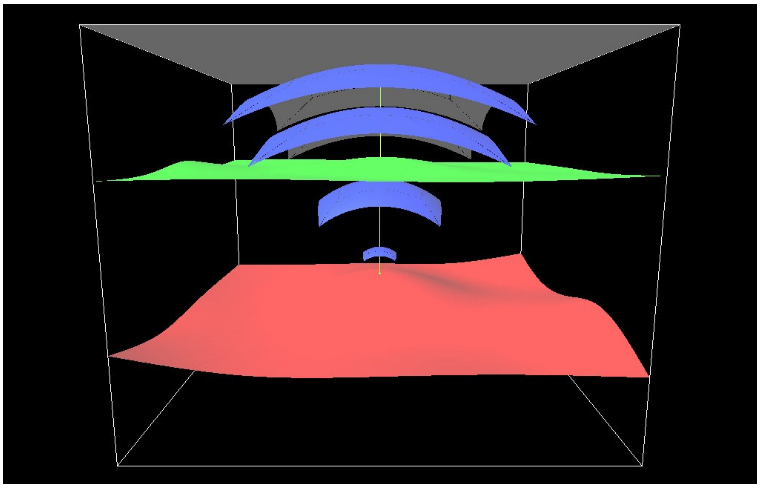

Figure 1: Subsurface NIP wave propagation. The wavefronts of this wave are presented by blue surfaces in different times. The normal (central) ray is in green.

NUMERIC RESULTS

The synthetic model used for the numeric tests in this work is shown in the figure 2. It is a simple 3-D acoustic model composed by two layers on a semi-space. Each layer is separate for curved and continuous surfaces. The wave propagation velocities are constant for each layer. The velocities from the top to the base are 1.7 km/s, 2.3 km/s and 3.6 km/s for the half-space.

Using the SW3D CRT software (Complete Ray Tracing) a seismic experiment was accomplished, where the receivers were dispersed in a regular mesh and with a located source in the mesh center. The lines in blue in the figure 2 are the rays of the primary reflections associated to the second interface. In figures 3 and 4 the blue surfaces are the traveltimes of the primary reflections corresponding to the second reflector. The eight parameters 3-D ZO-CRS were calculated with a ray tracing program (Höcht, 2002) for the 3-D model considered (figure 2). Later, using the approximations (7) and (10) we calculate, respectively, the paraxial ray traveltimes associated to a reflected and diffracted central ray. In the figure 3 the red color surface was calculated with (7), known as 3-D ZO-CRS operator for being associated to a reflected central ray. In a similar way, in the figure 4, the green color surface was calculated with (10). Due the fact of these traveltimes be associated with a diffracted central ray, this surface can be called of common diffraction surface or 3-D ZO-CDS. The comparison of these approximate surfaces formed by paraxial traveltimes with the surface of the true times of primary reflections, it reveals that the 3-D ZO-CRS operator associate to a reflected central ray has better adjustment if compared with the 3-D ZO-CDS operator, associate to a diffracted central ray.

CONCLUSIONS

In this work a theoretical revision was presented on the wavefront curvatures propagation laws in three dimensions. With a 3-D ray tracing program the eight NIP and N wavefront attributes were calculated. We Also made a revision of the 3-D traveltime hyperbolic approximations , with the objective of calculating the paraxial ray traveltimes to a central ray with Zero Offset.

As preliminaries numeric results in the current model 3-D two surfaces were presented, formed by paraxial ray traveltimes, an associated to a reflected central ray (3-D ZO-CRS), and the other associated to a diffracted central ray (3-D ZO-CDS).

The comparison of the surfaces or 3-D ZO-CRS and 3-D ZO-CDS operators with the surface of the exact traveltimes reveal that the operator 3-D ZO-CRS has better adjustment with the exact times, compared with the 3-D ZO-CDS operator. However, the 3-D ZO-CRS operator can also be used for the 3-D seismic stack.

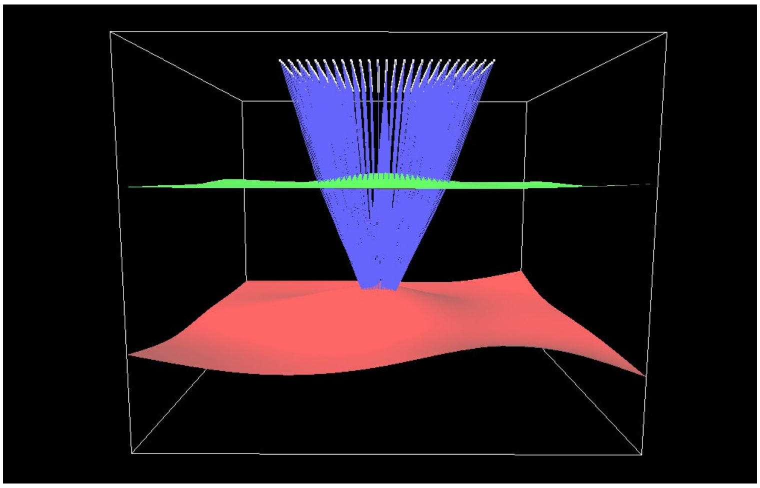

Figure 2: 3-D synthetic model representation. This model is formed by two layers on a semi-space. See the geometry acquisition in the measurement surface. The rays are in blue.

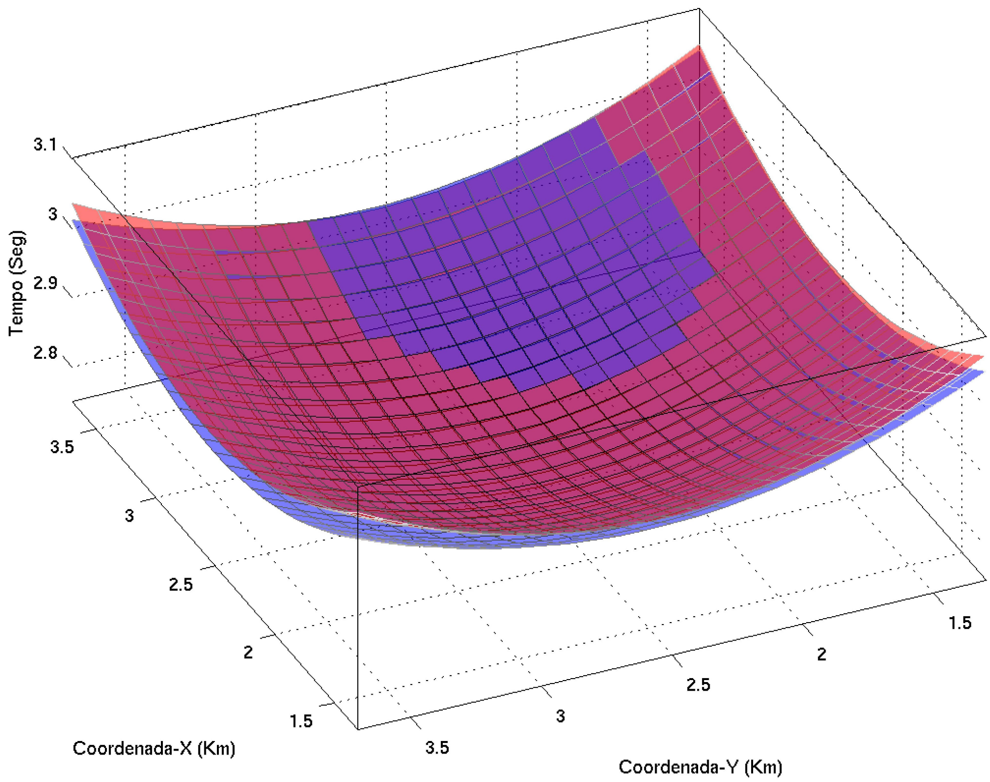

Figure 3: True (blue color) and approximated (red color) traveltime surfaces of primary reflections associated with the second reflector. The red surface was obtained considering a reflected central ray.

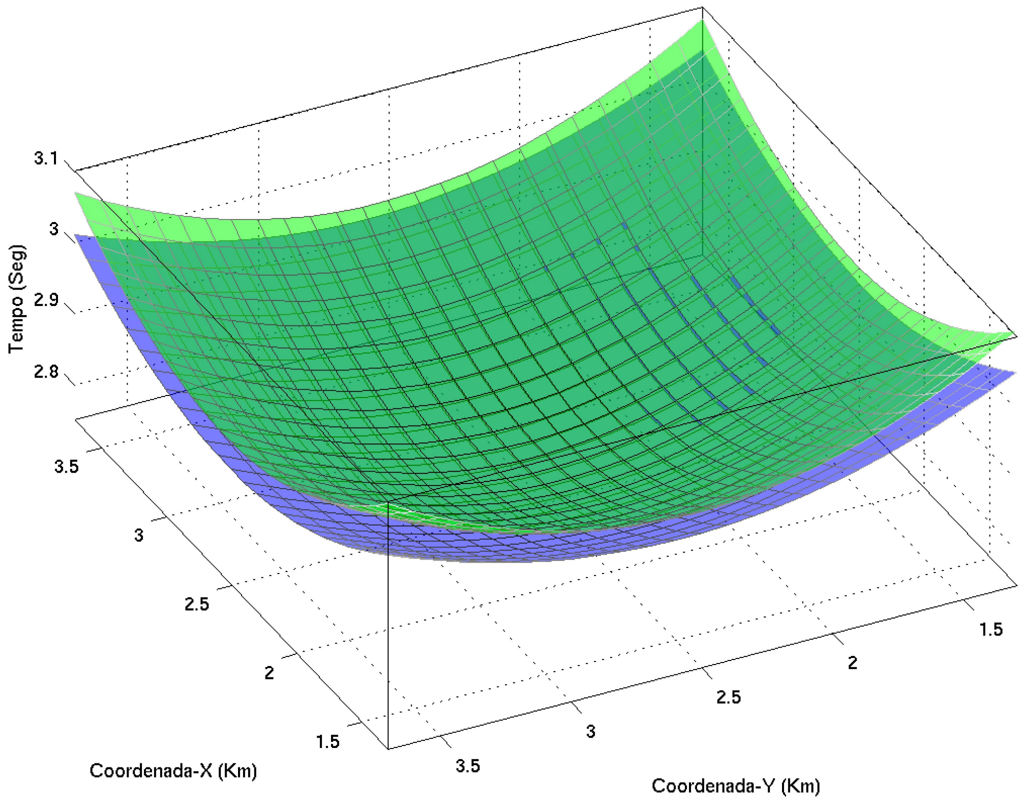

Figure 4: True (blue color) and approximated (green color) traveltime surfaces of primary reflections associated with the second reflector. The green surface was obtained considering a diffracted central ray.

ACKNOWLEDGMENTS

This work was kindly supported by the sponsors of the *Wave Inversion Technology (WIT) Consortium*, Karlsruhe, Germany.

REFERENCES

Bergler, S., Hubral, P., Marchetti, P., Cristini, A., and Cardone, G. (2002). 3D common-reflection-surface stack and kinematic wavefield attributes. *The Leading Edge*, 21:1010–1015.

Cardone, G., Cristini, A., Bonomi, E., Marchetti, P., Zambonini, R., Hubral, P., and Mann, J. (2003). 3D zero-offset CRS for narrow-azimuth data: formulation and examples. *In EAGE/SEG Summer Research Workshop on processing and imaging of seismic data - using explicit or implicit velocity model information? EAGE. Expanded Abstract, Sesion T006*.

Cerveny, V. and Ravindra, R. (1971). Theory of seismic headwaves. *Toronto, University of Toronto Press*.

Chira, P. (2003). Empilhamento pelo método Superfície de Reflexão Comum 2-D com Topografia e Introdução ao caso 3-D (in portuguese). *Ph.D. Thesis, Federal University of Para, Brazil*.

Chira-Oliva, P., Cruz, J. C. R., Hubral, P., and Tygel, M. (2003). Theoretical aspects of the 3-D ZO CRS stack. *8th International Congress of the SBGf, BRAZIL*.

Cristini, A., Cardone, G., Chira, P., Hubral, P., and Marchetti, P. (2001). 3D Zero-Offset Common Reflection Surface Stack for Land Data. *46th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, Session W5-13*.

Cristini, A., Cardone, G., and Marchetti, P. (2002). 3D Zero-Offset Common Reflection Surface Stack for land data - real data example. *Expanded Abstract of the 64th EAGE Conference and Technical Exhibition, Sesion B15*.

Höcht, G. (2002). Traveltime approximations for 2D and 3D media and kinematic wavefield attributes. *PhD thesis. University of Karlsruhe, Germany*.

Hubral, P. (1983). Computing true amplitude reflections in a laterally inhomogeneous earth. *Geophysics*, 48:1051–1062.

Hubral, P. and Krey, T. (1980). Interval velocities from seismic reflection traveltime measurements. *Society of Exploration Geophysicists, Tulsa, OK,203pp*.

Jäger, R. (1999). The common-reflection surface stack - theory and application. *Master's thesis, University of Karlsruhe, Germany*.