

# FAST ESTIMATION OF CRS PARAMETERS USING LOCAL SLOPES

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## ABSTRACT

*The complete set of CRS parameters can be extracted from seismic data by an application of modern local-slope-extraction techniques. The necessary information about the CRS parameters is contained in the slopes of the common-midpoint and common-offset sections at the central point. As demonstrated by a synthetic data example, the slope extraction is sufficiently robust to allow for derivation of the extracted slope field. This enables the calculation of the CRS parameters from the extracted slopes and their derivatives. In this way, the CRS parameter extraction can be sped up by several orders of magnitude.*

## INTRODUCTION

Present-day techniques to estimate the traveltimes parameters of the common-reflection-surface (CRS) stack rely on local coherence analyses that are tedious and time-consuming processes (see, e.g., Jäger et al., 2001; Hertweck et al., 2007). However, the extraction of traveltimes attributes, particularly local slopes, has received strong attention in the recent past, because local slopes directly extracted from prestack data are useful in a variety of seismic imaging processes. Perhaps, the most visible ones are those connected with seismic tomography, in which not only traveltimes but also slownesses of events and possible other time-domain attributes are used to build a velocity model. Most prominent examples are slope tomography (Sword, 1987; Biondi, 1990), stereotomography (Billette and Lambaré, 1998; Billette et al., 2003) and normal-incidence-point (NIP) wave tomography (Duvencok, 2004).

Recently, Fomel (2002) presented fast techniques how to extract local slopes and even curvature related traveltimes parameters. In this paper, we show how the CRS parameters relate to local slopes in order to speed up their extraction. Of course, this relationship is most straightforward for the emergence angle of the normal ray,  $\beta$ , since this parameter is nothing else but a local slope.

The conventional procedure to extract this slope parameter relies on local slant stacks (Ottolini, 1983a). In this method, a local coherence analysis is carried out at each point in the seismic section along short straight-line elements in all possible directions. The direction with the highest coherence defines the slope value at that point.

However, the local-slant-stack approach to local-slope extraction has a number of drawbacks. First of all, the method has a high computational cost. Since the space of local slopes must be closely sampled, there is a high number of coherence analyses to be carried out. The second drawback lies in the method's sensitivity to the aperture of the local slant stacks. An adequate aperture is problem dependent and thus hard to know in advance. Finally, as demonstrated by Schleicher et al. (2007), the extracted slopes are not always reliable, but can be biased towards too high dips.

For this reason, Schleicher et al. (2007) studied alternative, faster and more reliable ways to extract local slopes. They showed that local slopes can be extracted faster and more reliably than using local slopes using modern extraction techniques. Therefore, the question arises whether these kind of techniques can be extended to allow for the extraction of the remaining CRS parameters.

In this work, we demonstrate that such an extension is possible. We show how the complete set of CRS parameters can be estimated by the application of modern, more advanced local-slope-extraction techniques, that are several orders of magnitude faster than conventional local coherence analysis.

### HYPERBOLIC MOVEOUTS

To understand the relationship between local slopes and the CRS parameters, let us start with a brief review of the underlying traveltimes approximations.

#### Common-midpoint traveltimes

In the 2D situation of a single (horizontal) seismic line and within a given common midpoint (CMP) gather, namely source-receivers pairs that are symmetrically located around a fixed central point  $x_0$ , the traveltimes function can be approximated by the *Normal Moveout (NMO)* function,

$$T_{CMP}(h) = \sqrt{T_0^2 + C h^2}. \quad (1)$$

In the above equation,  $T_{CMP}$  is the traveltimes from the source to the reflector and back to the receiver,  $h$  is the source-receiver half-offset, and  $T_0 = T_{CMP}(0)$  is interpreted as the CMP zero-offset (ZO) traveltimes, i.e., the traveltimes along the normal ray that emerges at the central point of the CMP gather. Moreover,  $C$  is a parameter related to the curvature of the traveltimes function. It is usually expressed as

$$C = C(x_0, T_0) = \frac{4}{V^2}, \quad (2)$$

where  $V$  can represent the stacking or NMO or rms velocity.

#### Common-reflection-surface traveltimes

The common reflection surface (CRS) method, introduced by Peter Hubral and co-workers (see, e.g., Hubral et al., 1998) represents a natural extension of the CMP method in two important aspects. Firstly, for each stacking trace location (above referred to as the central point of the CMP gather, now called simply the central point), the CRS method considers a supergather of source-receiver pairs, arbitrarily located with respect to the central point. In other words, the gather is not restricted to the CMP condition. Secondly, not only the NMO velocity, but also other additional parameters describe the traveltimes function and can thus be extracted from the data. In the 2D situation, studied in this work, three parameters are determined for each central point and all ZO traveltimes samples. The procedure is performed automatically, with no *a priori* selection of traveltimes samples.

To be able to stack source-receiver pairs that do not conform to the CMP condition, the CRS method utilizes the (generalized) hyperbolic moveout,

$$T_{CRS}(x, h) = \sqrt{[T_0 + A(x - x_0)]^2 + B(x - x_0)^2 + C h^2}, \quad (3)$$

where  $x$  and  $h$  denote the midpoint and half-offset coordinates of the source and receiver pair,  $x_0$  is the midpoint coordinate of the central point, and  $T_0 = T_{CRS}(x_0, 0)$  is the ZO traveltimes at the central point. As shown in Hubral et al. (1998), the parameters  $A$ ,  $B$  and  $C$  are related to physical quantities  $\beta$ ,  $K_N$  and  $K_{NIP}$ , referred to as the CRS parameters or attributes, by the relationships

$$A = \frac{2 \sin \beta}{v_0}, \quad B = \frac{2T_0 \cos^2 \beta}{v_0} K_N, \quad \text{and} \quad C = \frac{2T_0 \cos^2 \beta}{v_0} K_{NIP} = \frac{4}{V^2}, \quad (4)$$

where  $v_0$  denotes the near-surface medium velocity at the central point. The physical meaning of these parameters are: Angle  $\beta$  is the emergence angle of the ZO ray with respect to the surface normal, and  $K_N$  and  $K_{NIP}$  are the curvatures of the N- and NIP-waves, respectively (see Hubral, 1983). All these quantities are evaluated at the central point. The N- and NIP-waves are fictitious waves that start at the reflection point of the ZO ray, which is called normal-incident-point or NIP, and propagate upwards along

the ZO ray with half the medium velocity. The NIP-wave starts with a point source at NIP, i.e., with a curvature radius of zero, while the N-wave starts with a wavefront that possesses the same curvature as the reflector at NIP.

Observe that formula (3) reduces to the normal-moveout equation (1) for source-receiver pairs in the CMP gather at the central point, i.e., for  $x = x_0$ . Another important observation is that parameters  $A$ ,  $B$ , and  $C$  depend on  $T_0$ . Finally, note that the original CRS parameters  $\beta$ ,  $K_N$ , and  $K_{NIP}$  can be calculated once all three parameters  $A$ ,  $B$ , and  $C$  are known.

### Common-offset traveltimes

The remaining two CRS parameters,  $A$  and  $B$  can be determined from a common-offset (CO) gather in the vicinity of  $x_0$ . This is advantageous over the conventional detection in the ZO gather, because the latter is only available after stack and its quality already depends on  $C$ , while CO gathers are available in the original data, thus permitting a completely independent determination of  $A$  and  $B$ .

For the case of a CO gather with a fixed  $h > 0$ , formula (3) reduces to

$$T_{CO}(x) = \sqrt{T_{CMP}(h)^2 + 2AT_0(x - x_0) + D(x - x_0)^2}, \quad (5)$$

where  $D = A^2 + B$  and  $T_{CMP}$  is the traveltime for the offset ray with midpoint at  $x_0$ . It is given in terms of the ZO traveltime by equation (1).

## LOCAL SLOPES AND THE CRS PARAMETERS

Let us now study how the local slopes, i.e., derivatives of the above traveltime functions, can be used to extract information about the CRS parameters. In the following discussion, we will use capital letters  $T$  when talking about theoretical traveltime expressions and lowercase letters  $t$  when talking about time coordinates in seismic sections.

### CMP traveltime

Under the usual assumption of a horizontally stratified (or small-dip) media, the ray parameter for the reflection ray in the CMP gather with fixed central point (midpoint)  $x_0$ , can be approximated by the traveltime slope (see, e.g., Castagna and Backus, 1993). The derivative of equation (1) with respect to source-receiver offset  $2h$  yields

$$p = \frac{1}{2} \frac{d}{dh} T_{CMP} = \frac{Ch}{2T_{CMP}}. \quad (6)$$

Thus, if we know the local slope  $p = p(h, t)$  at a point  $(h, t)$  in a CMP gather, we can use equations (1) and (6) to eliminate  $C$  from the moveout equation. This yields the NMO coordinate map

$$t_0 = \sqrt{t^2 - 2htp(h, t)}. \quad (7)$$

which describes the relationship between the time coordinates  $t$  in a CMP section and  $t_0$  of a ZO trace at  $x_0$ . Equation (7) can be immediately used for an automatic NMO correction (Ottolini, 1983b; Schleicher et al., 2007), since it tells us how to move a pixel of information from coordinates  $(h, t)$  in the CMP section to  $(h, t_0)$  in the NMO corrected section.

On the other hand, equation (6) provides us with the first relationship between a curvature parameter and a local slope. Rewriting equation (6), we see that parameter  $C$  at half-offset  $h$  and ZO time  $t_0$  is given by

$$C(h, t_0) = \frac{2tp(h, t)}{h}. \quad (8)$$

From equation (8), we infer the procedure that achieves the determination of parameter  $C$ . All that needs to be done is to correct the extracted local slope  $p$  at a position  $(h, t)$  in the CMP section with the factor  $2t/h$  and then to transfer the obtained value of  $C$  to the  $C$ -parameter section at  $(x_0, t_0)$ . This can be done fully automatically. Since there is redundant information from all half-offsets, the final  $C(x_0, t_0)$  can be calculated by averaging over all  $C(h, t_0)$  at  $x_0$ .

### CO traveltimes

A similar scheme can be applied to extract the remaining CRS parameters from a CO gather with fixed  $h > 0$  in the vicinity of the central point  $x_0$ . From traveltimes formula (5), we see that the local slope  $q$  at a point  $(x, t)$  in the CO gather is given by

$$q = \frac{d}{dx} T_{CO} = \frac{A T_0 + D (x - x_0)}{T_{CO}}. \quad (9)$$

Let us assume that the local slope  $q = q(x, t)$  is known at all positions  $(x, t)$  in the CO gather. Taking the derivative of equation (9) with respect to  $x$ , and recombining the resulting equation for  $q_x = dq/dx$  with equation (9), we arrive at

$$D(x, t) = q(x, t)^2 + t q_x(x, t). \quad (10)$$

Equation (10) tells us how to determine the value of the combined parameter  $D$  of equation (5) at a certain point  $(x, t)$  in the CO section. However, this value of  $D$  is still associated with a point  $(x, t)$  in the CO section and needs to be attributed to its respective coordinate  $t_0$  in the ZO section to be constructed. This  $t_0$  is yet unknown and must be determined.

For this purpose we solve equation (9) for  $A$  and substitute the resulting expression in equation (5). Solving for the CMP traveltimes, we obtain the coordinate map

$$t_{CMP} = \sqrt{t^2 - 2 q(x, t) t (x - x_0) + D (x - x_0)^2}, \quad (11)$$

between the CO and CMP sections. Map (11) can be executed once  $q$  and  $q_x$  have been detected at every point  $(x, t)$  in the CO section.

As a last step, the time coordinate  $t_{CMP}$  must be related to its associated ZO traveltimes  $t_0$ . This can be done using formula (7), i.e.,

$$t_0 = \sqrt{t_{CMP}^2 - 2 h t_{CMP} p(h, t_{CMP})}. \quad (12)$$

Combining the above equations and coordinate maps, we can construct a  $D(x_0, t_0)$  section. This section, in turn, allows to construct the corresponding sections for the parameters  $A$  and  $B$  at time  $t_0$ . Parameter  $A$  is given by

$$A(x_0, t_0) = \frac{q(h, t) t - D(x_0, t_0) (x - x_0)}{t_0}, \quad (13)$$

and parameter  $B$  reads

$$B(x_0, t_0) = D(x_0, t_0) - A(x_0, t_0)^2. \quad (14)$$

This mapping procedure can be applied for different CO sections around the central point  $x_0$ . As in the parameter  $C$  case, the values of  $A$  and  $B$  are transferred to the respective parameter sections at  $(x_0, t_0)$ . The final values of  $A$  and  $B$  are obtained by averaging over all values that are attributed to the same point  $(x_0, t_0)$ .

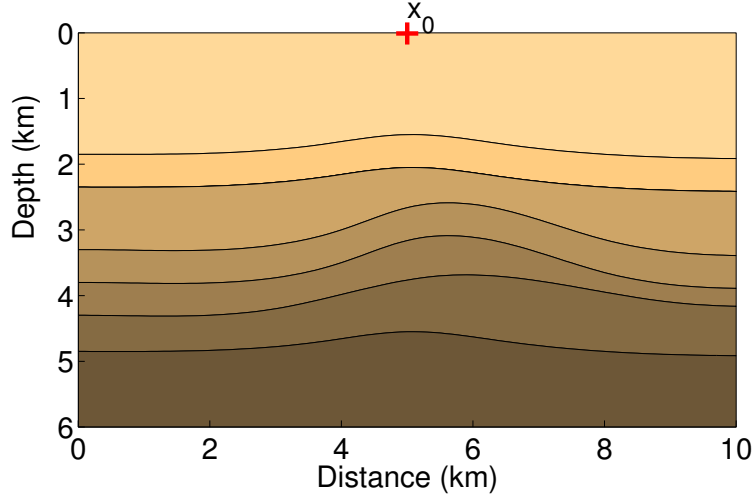
### EXTRACTING THE LOCAL SLOPES

The extraction of local slopes is done by so-called plane-wave destructors. The differential equation that describes a local plane-wave event in a seismic section is given by (Claerbout, 2004)

$$\psi_y(y, t) + s \psi_t(y, t) = 0, \quad (15)$$

where  $\psi(y, t)$  is the wavefield,  $t$  is the time coordinate, and  $y$  is the horizontal coordinate, i.e., offset ( $2h$ ) in the case of a CMP section or midpoint ( $x$ ) in the case of a CO section. Quantity  $s$  represents the local slope (i.e.,  $p$  or  $q$  in a CMP or CO section), which may depend on  $y$  and even on  $t$ , i.e., generally,  $s = s(y, t)$ .

To extract the slopes, we use basically the technique presented in Claerbout (2004) and Schleicher et al. (2007). We just employ a few small alterations in the implementation that we found to turn the process more stable.



**Figure 1:** Synthetic model for the experiments.

For each pair  $(\xi, \tau)$  in the seismic section, we select a small window of points  $y_i, t_j$  with  $(i, j) \in W$ . Let  $\Psi_y(y_i, t_j)/\Delta y$  and  $\Psi_t(y_i, t_j)/\Delta t$  be the discretized values for the derivatives  $\psi_y$  and  $\psi_t$ , respectively, in the selected window, with  $\Delta y$  and  $\Delta t$  being the respective sample sizes. In order to accomplish with equation (15), we minimize the quadratic residual

$$R(s) = \sum_{(i,j) \in W} \left[ \Psi_y(y_i, t_j) + s \frac{\Delta y}{\Delta t} \Psi_t(y_i, t_j) \right]^2. \quad (16)$$

The solution is easily found as

$$s^* = s^*(\xi, \tau) = -\frac{\Delta t}{\Delta y} \frac{\sum_{(i,j) \in W} \Psi_y(y_i, t_j) \Psi_t(y_i, t_j)}{\sum_{(i,j) \in W} \Psi_t^2(y_i, t_j)}, \quad (17)$$

where  $(\xi, \tau)$  is the center of the selected window. One measure for the fit is given by the normalized residual. To avoid the problems reported in Schleicher et al. (2007) at local slopes close to zero, we use the slightly modified coherence measure

$$E(\xi, \tau) = 1 - \frac{R(s^*)}{\sum_{(i,j) \in W} [\Psi_y^2(y_i, t_j) + \Psi_t^2(y_i, t_j)]}. \quad (18)$$

### SYNTHETIC EXAMPLES

For the numerical experiments we used a simple synthetic model of a stratified medium with homogeneous layers as shown in Figure 1, with a fixed central point at  $x_0 = 5$  km. We simulated a CMP section with 70 half-offsets ranging from  $-1$  km to  $1$  km, and a single CO section with constant half-offset of  $250$  m and 51 midpoints ranging from  $4.5$  km to  $5.5$  km. In both simulations the time sample was  $4$  ms. For better control over the CRS parameters, we simulated all reflection events as if the velocities in the model were constant rms velocities down to the corresponding reflector. The rms velocity for the shallowest reflector was  $4.0$  km/s, increasing by  $0.2$  km/s for each layer to  $5.0$  km/s for the deepest one. Also shown in Figure 1 is the point  $x_0$  for which we carry out the analysis.

Since real data consist of many CO gathers for different offsets, these even could be used to improve redundancy. However, since the traveltimes approximations are valid in the vicinity of the normal ray, too far offsets should be avoided. In our numerical experiments, best results were achieved by using only one or two small offsets.

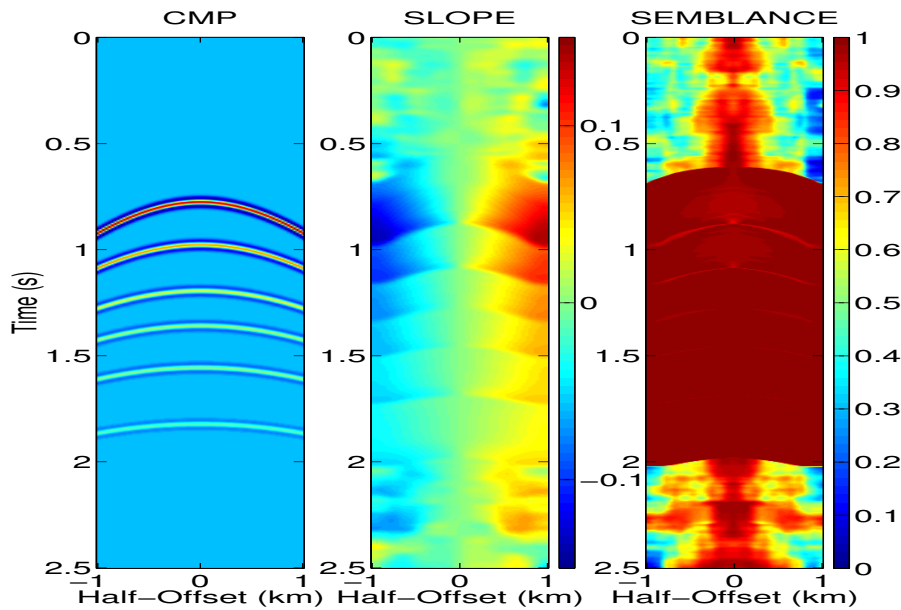


Figure 2: Experiment without noise: CMP, slope and semblance sections.

#### Noise-free data

The results of the CRS parameter extraction for noise-free data are shown in Figures 2 to 4. In Figure 2 we see the CMP section and the corresponding slope ( $p$ ) and semblance ( $E$ ) sections as obtained using equations (17) and (18). The extraction window used 21 traces and 41 time samples. The extracted slopes values correspond very well to the expected values. The semblance is very high in the time interval of the events, indicating high reliability of the extracted slope values.

Figure 3 depicts the corresponding CO section together with its extracted slope ( $q$ ) and semblance sections. Again, the extracted slope values correspond perfectly to the dips in the CO section, and the high semblance in the time interval of the events indicates high reliability of the extracted slope values.

The values of the CRS parameters  $A$ ,  $B$  and  $C$  at the chosen point  $x_0$  (only one trace) were then determined from these extracted slopes using equations (8), (13) and (14). The results are depicted in Figure 4 (red lines), together with the respective exact values (blue crosses). Of course, true values are only available at the reflection events while the extraction procedure yields values at all times. Figure 4 also shows the values of the accumulated semblances ( $S$ ) for  $p$  and  $q$ . These are the mean values of the semblances of all values of the parameters  $A$ ,  $B$  and  $C$  that contributed to the final values. As usual for noise-free data, the semblance is very high everywhere.

The overall quality of the so extracted CRS parameters is more than satisfying. Parameters  $C$  and  $A$  (related to  $K_{NIP}$  and  $\beta$ ) have been determined near perfectly. Parameter  $B$  (related to  $K_N$ ), known to be the most unstable parameter, has not been recovered with the same precision. However, the extracted values show the general trend and only slightly underestimate the true values.

Another way of evaluating the quality of the extracted CRS parameters is by seeing how well the corresponding traveltime approximations fit the data Figure 5 shows this analysis. We see that the CMP and CO reflection events are perfectly fitted by traveltimes (1) and (5) using the extracted parameters (red lines), confirming their high quality. The blue lines representing the traveltime curves with the exact parameter values are invisible because they are entirely covered by the red lines.

#### Noisy data

For a more realistic test of the proposed extraction method, it is important to investigate how it reacts to the presence of noise. Therefore, we have repeated the above experiment after 30% adding random noise to the synthetic data. In other words, the noise level was chosen such that the signal-to-noise ratio for

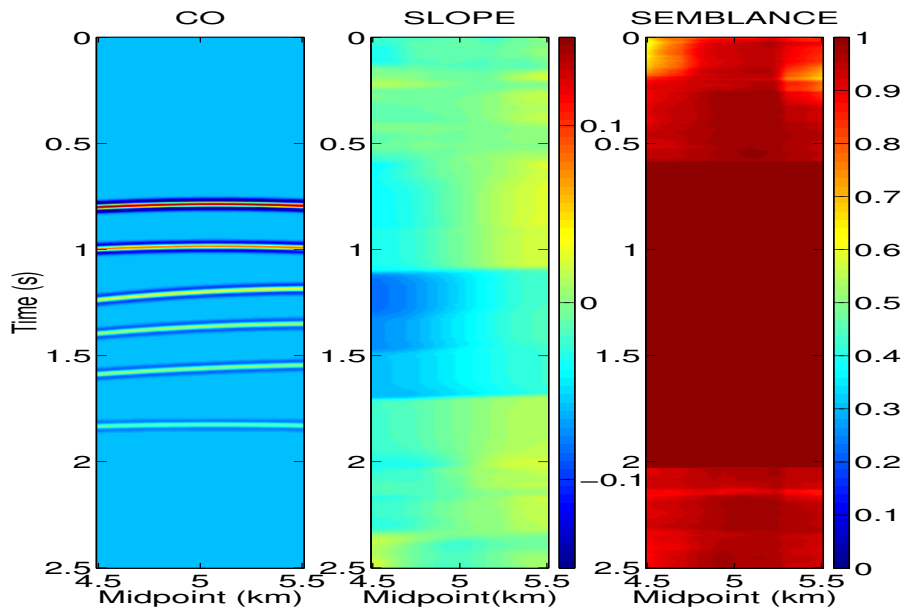


Figure 3: Experiment without noise: CO, slope and semblance sections.

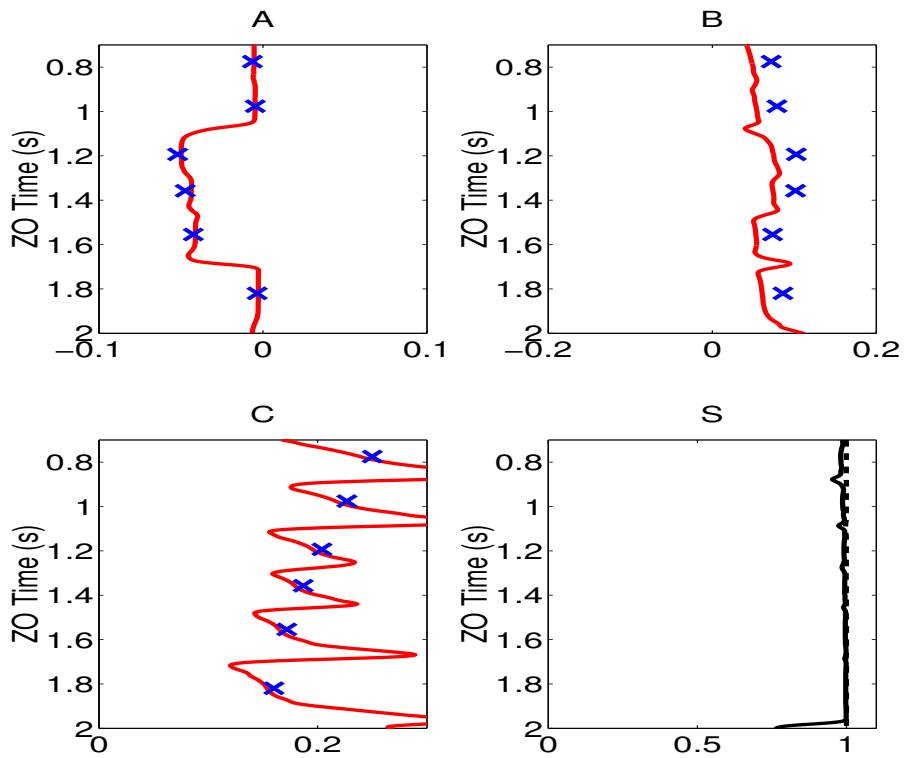
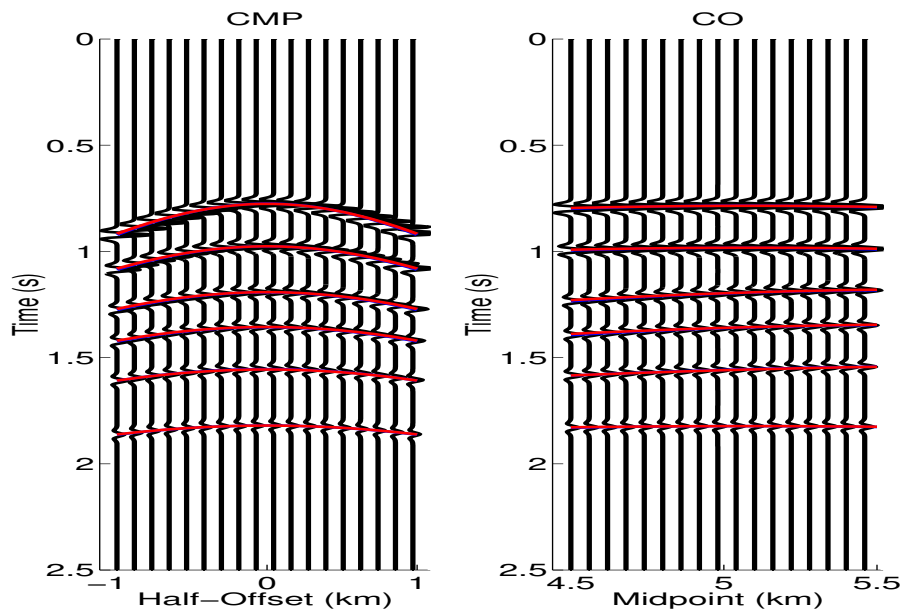
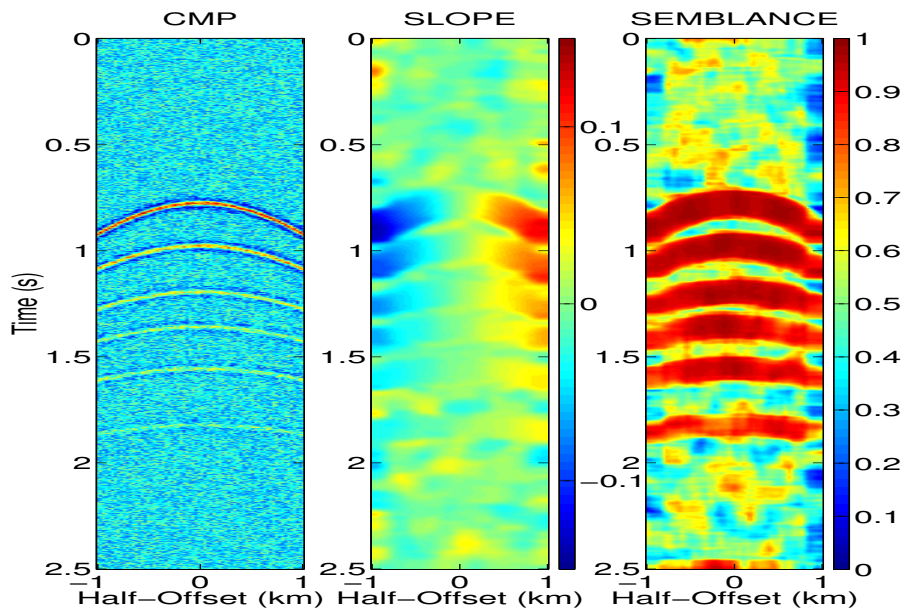


Figure 4: Experiment without noise: CRS parameters (A, B, C) and semblances (S). The exact are the blue crosses and the estimated ones are the red lines.

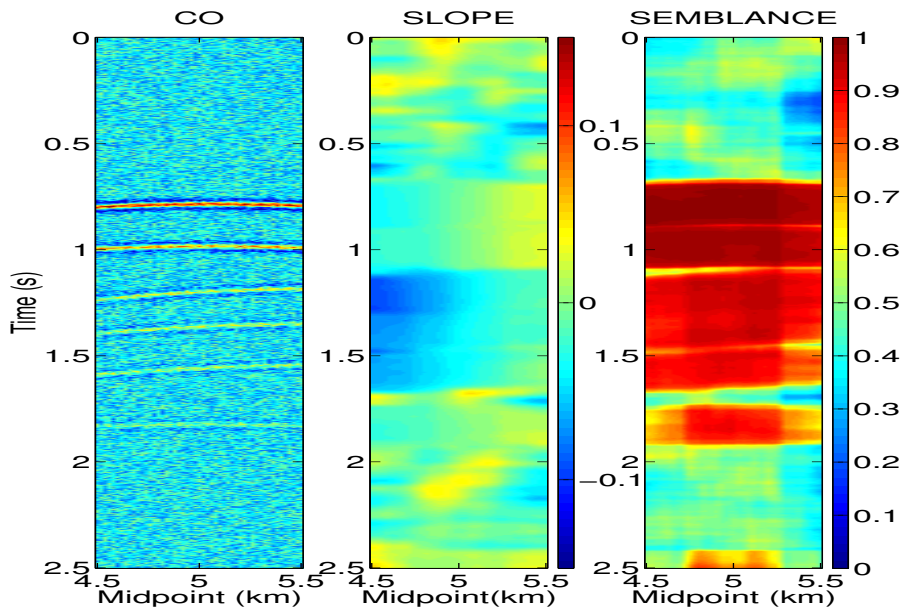


**Figure 5:** Experiment without noise: CMP and CO sections with exact (blue) and estimated (red) travel-times.



**Figure 6:** Experiment with 30% added noise: CMP, slope and semblance sections.





**Figure 7:** Experiment with 30% added noise: CO, slope and semblance sections.

the strongest arrival, the reflection from the topmost reflector, was about 3. This resulted in a signal-to-noise for the weakest arrival, the reflection from the deepest reflector, of about 1. Figures 6 to 8 show the extraction results for the noisy data. The general behavior of the slope and semblance sections of the CMP and CO gathers (Figures 6 and 7) is very similar to the corresponding sections for the noise-free situation (Figures 2 and 3), demonstrating that the estimation of local slopes is sufficiently stable at this noise level.

Figure 8 compares the extracted parameter values (red lines) to the exact ones (blue crosses). The results are still satisfactory. Parameters  $A$  and  $C$  have lost almost no quality in comparison to the noise-free situation (Figure 4). The noise seems to influence mainly the extracted values of parameter  $B$ , the quality of which has further deteriorated. It is to be noted that for the noisy data, the accumulated semblance  $S$  exhibits significant peaks at the actual reflection times, thus carrying useful information on where the extracted parameter values are actually reliable.

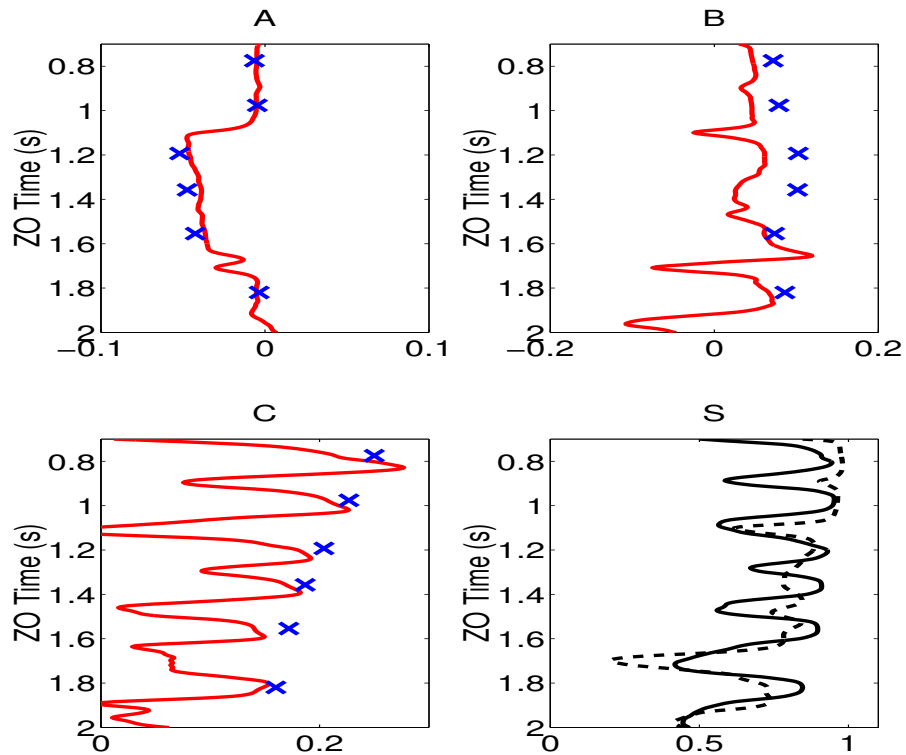
The fit achieved with these extracted parameters is evaluated in Figure 9. We see that the small errors in parameter  $C$  already lead to small deviations between the exact and fitted traveltimes curves (blue and red lines, respectively) in the CMP gather. On the other hand, the larger errors in  $B$  lead to almost no deviation between the exact and fitted traveltimes curves in the CO gather. This behavior is the reason why it is almost impossible by any method to extract reliable values for  $B$ .

These numerical experiments demonstrate that the estimation of CRS parameters from local slopes is sufficiently stable to permit their automatic extraction, even for noisy data. We stress again that this extraction procedure is orders of magnitude faster than the conventional method by local coherence analyses.

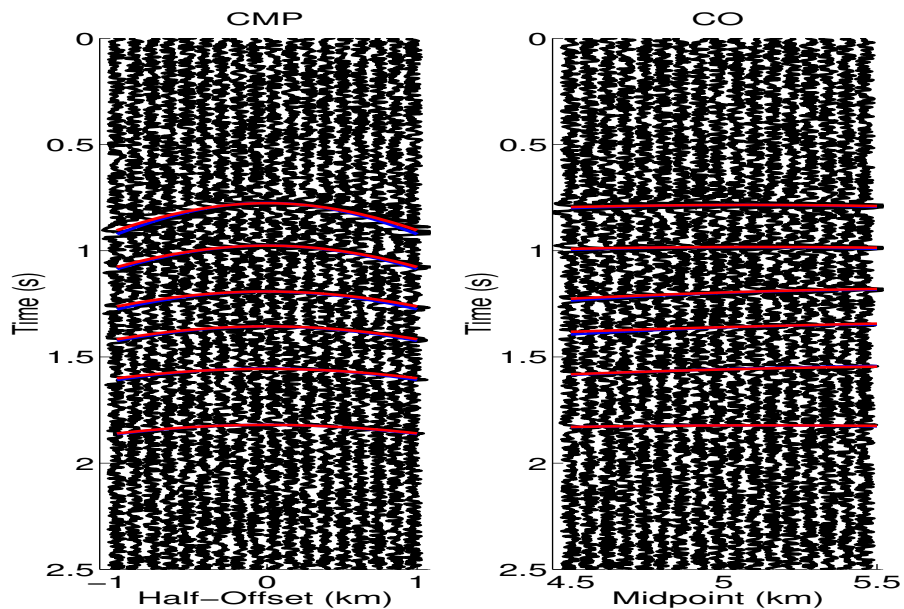
## CONCLUSIONS

CRS parameter extraction by local coherence analysis has a number of drawbacks. First and most important of all, the method has a high computational cost. Since the space of possible parameter values must be closely sampled, there is a high number of coherence analyses to be carried out. The second drawback lies in the method's sensitivity to the aperture of the local stacking operators. An adequate aperture is problem dependent and thus hard to know in advance.

In this work, we have shown that it is possible to overcome these problems by a different approach to parameter extraction. We have presented an application of modern local-slope-extraction techniques so as to allow for the detection of the complete set of CRS parameters. The necessary information about the CRS parameters is contained in the slopes of the common-midpoint and common-offset sections at the central



**Figure 8:** Experiment with 30% added noise: CRS parameters (*A*, *B*, *C*) and semblances (*S*). The exact are the blue crosses and the estimated ones are the red lines.



**Figure 9:** Experiment with 30% added noise: CMP and CO sections with exact (blue) and estimated (red) traveltimes.

point. As demonstrated by a synthetic data example, the slope extraction is sufficiently robust to allow for derivation of the extracted slope field. This enables the calculation of the CRS parameters from the extracted slopes and their derivatives. In this way, the CRS parameter extraction can be sped up by several orders of magnitude.

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