MIGRATION VELOCITY ANALYSIS BY DOUBLE PATH-INTEGRAL MIGRATION

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ABSTRACT

The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocity models. Those velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. In this way, the overall image forms with no knowledge of the true velocity model. However, the velocity information associated with the final image can be determined in the process. By executing the path-integral imaging twice, weighting one of the stacks with the velocity value, the stationary velocities that produce the final image can then be extracted by a division of the two images. A numerical example demonstrates that quantitative information about the migration velocity model can be determined by double path-integral migration.

INTRODUCTION

The quality of seismic images of the earth's interior is strongly dependent on the available velocity model. Keydar (2004) and Landa (2004) have proposed a path-integral approach to seismic imaging in order to overcome this dependency on the knowledge of a velocity model. The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocity models. Those velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. In this way, the overall image forms with no need to know the true velocity model. This new imaging approach resembles in a certain way the principles of the Feynman path integral (Feynman and Hibbs, 1965). It allows to replace the complex optimization problem of estimating an adequate migration velocity model by "integration along all possible trajectories," that is, by a summation over the images for all possible migration velocity models.

The path-summation method has recently enjoyed renewed attention in seismics. It was used for obtaining approximate waveform solutions to the scalar wave equation (Schlottmann, 1999; Lomax, 1999). The path-summation method constructs an approximate wavefield by summation over the contributions of elementary signals propagated along a representative sample of all possible paths between the source and observation points.

Keydar (2004) applied the technique to inversion by homeomorphic imaging, which is based on an NMO correction formula represented as a function of certain wavefront parameters (radii of curvature and emergence angle), similar to the CRS method. By changing the wavefront parameters, the NMO time correction curve changes its position. Instead of determining optimal stacking parameters, Keydar (2004) proposed to sum along all possible NMO time curves. Due to stationarity principles, the prevailing contributions are from only those time curves that are nearly in phase. Contributions from summing along the remaining NMO curves cancel each other because the phase oscillates rapidly between positive and negative values.

Landa (2004) extended the idea to time migration. By analogy to the use of Feynman's path integrals in waveform modelling, he proposed to obtain the subsurface seismic image by a summation of seismic signals over a representative sample of all possible paths/trajectories between a source and observation point. In his approach, the velocity model is assumed to be unknown and the summation trajectories are defined in the time (data) domain rather than in the depth (model) domain. For zero-offset (post-stack) migration, the path-summation imaging consists of a summation of seismic prestack data along all possible stacking hyperbolae instead of only along a subset, corresponding to highest coherency criteria (e.g., semblance) in the conventional zero-offset imaging (stack, Multifocusing, CRS). For full pre-stack time migration (PSTM), path-summation imaging consists of a summation of elementary signals over all possible diffraction curves instead of only along a subset, corresponding to the chosen migration velocity. The constructive and destructive interference of the elementary signals contributed by each path/trajectory produces an image that converges towards the correct one which would be obtained by a migration using the correct velocity. First applications of path-summation imaging in depth migration were presented by Landa et al. (2005) and Shtivelman and Keydar (2005). A similar idea was recently used by Anikiev et al. (2007) to locate seismic events in an unknown velocity field.

Landa et al. (2006) discuss path-summation imaging in more conceptual and theoretical detail. They stress that there are three essential conditions for path-summation imaging to be successful:

1. the argument of the path integral is chosen adequately;

2. the integration is carried out over a representative sample of all possible trajectories;

3. a properly designed weight function is applied in the multipath summation.

The first condition takes care that the path-integral description actually does what it is supposed to do, i.e., enhance stationary contributions and cancel anything else. The second condition guarantees the completeness of the model space, making sure that interference actually can do its job, constructively enhancing desired features of the image and destructively reducing undesired ones. The third condition makes up for deficiencies in the second one, since however fine we sample the complete continuous model space, there is no way of covering it completely. A successful weight function was discussed by (Keydar et al., 2008).

The beauty of the multipath summation method is that it eliminates the need to construct a migration velocity model before imaging. The multipath stack itself takes care of enhancing the true image as the only one that interferes constructively with images from slightly perturbed models. However, this very beauty turns into a drawback when the actual velocity model that is associated with the resulting image is needed, as is the case in many seismic applications. In this paper, we show how the multipath summation can be modified to extract a meaningful velocity model together with the final image. By executing the path-integral imaging twice and weighting one of the stacks with the used velocity value, the stationary velocities that produce the final image can then be extracted by a division of the two images. A numerical example demonstrates that information about the migration velocity can be extracted successfully from path-integral migration.

MULTIPATH-SUMMATION TIME MIGRATION

In the notation of Landa et al. (2006), the multipath-summation time-migration operator can be written as

$$V_W(\mathbf{x}) = \int d\alpha \; w(\mathbf{x},\alpha) \int d\xi \int dt \; U(t,\xi)\delta(t - t_d(\xi,\mathbf{x};\alpha)) \; , \tag{1}$$

where V_W is the resulting time-migrated image at an image point with coordinates $\mathbf{x} = (x,\tau)$, x being lateral distance and τ vertical time. In integral (1), $U(t,\xi)$ denotes a seismic trace at coordinate ξ in the seismic data, and $t_d(\xi,\mathbf{x};\alpha)$ is a set of stacking surfaces corresponding to a set of possible velocity models α. Note that generally, the migration velocity α is a function of the position $\mathbf{x}$ of the image point, i.e., $\alpha = \alpha(\mathbf{x})$. The integration is weighted by a weight function $w(\mathbf{x},\alpha)$, which is designed to attenuate contributions from unlikely trajectories and emphasize contributions from trajectories close to the optimal one. There are several possible choices for $w(\mathbf{x},\alpha)$. We opted for an exponential weight function of the form

$$w(\mathbf{x},\alpha) = \exp[-P(\mathbf{x};\alpha)/\sigma^2] \; , \tag{2}$$

where $P(\mathbf{x};\alpha)$ is the squared average of the absolute value of the local event slopes in the common-image gather (CIG) at $\mathbf{x}$. The local event slopes are estimated using corrected least-square plane-wave filters as described in Schleicher et al. (2007). Parameter σ adjusts the half-width of the Gaussian bell function away from the desired events with $P = 0$. In our implementation, we chose $\sigma = 0.1\Delta\tau/\Delta x$. Since weight (2) is a real-valued exponential weight function, the path integral in our implementation is of the Einstein-Smoluchovsky type (Landa et al., 2006). This choice of the weight function guarantees condition 3 above to be satisfied.

According to Laplace's method (see, e.g., Erdélyi, 1956), integrals of the form of equation (1) with an exponential weight of the type of equation (2) have their main contribution from the point α_0 at which the function in the exponent has its maximum value. Clearly, in our case, the maximum value is reached at $P = 0$. Hence, the stationary value α_0 corresponds to the best possible migration velocity. Using Laplace's method, integral (1) can be asymptotically evaluated to yield

$$V_W(\mathbf{x}) \approx \sqrt{\frac{2\pi\sigma^2}{P''(\alpha_0)}} Q_0(\mathbf{x};\alpha_0) \; , \tag{3}$$

where $P''(\alpha_0)$ denotes the second derivative of the squared local slope mean P as a function of the varying migration velocity α. Moreover, $Q_0(\mathbf{x};\alpha_0)$ denotes the desired migration result with the stationary migration velocity α_0 [see also Landa et al. (2006)], viz.,

$$Q_0(\mathbf{x};\alpha_0) = \int d\xi \int dt \; U(t,\xi)\delta(t - t_d(\xi,\mathbf{x};\alpha_0)) \; . \tag{4}$$

Equation (3) justifies the claim that the result of a multipath summation produces a migrated image. In fact, we see that the result of multipath summation is directly proportional to the desired migration result.

Double multipath summation

The observation that the summation result is proportional to the desired image has another important consequence. It implies that the use of a slightly modified weight function,

$$\tilde{w}(\mathbf{x},\alpha) = \alpha \exp(-P((\mathbf{x},\alpha)/\sigma^2) \; , \tag{5}$$

will lead to a slightly modified migration result,

$$\tilde{V}_W(\mathbf{x}) \approx \alpha_0 \sqrt{\frac{2\pi\sigma^2}{P''(\alpha_0)}} Q_0(\alpha_0) \; . \tag{6}$$

In other words, results (3) and (6) differ only by a constant factor, this factor being the true migration velocity at $\mathbf{x}$.

This readily suggests that the migration velocity can be extracted from such a procedure by simply dividing the two migration results (3) and (6), i.e.,

$$\alpha_0 \approx \frac{\tilde{V}_W(\mathbf{x})}{V_W(\mathbf{x})} \; . \tag{7}$$

In fact, this idea of extracting quantities from multiple stacks is not new but has already been previously discussed in the framework of Kirchhoff migration (Bleistein, 1987; Tygel et al., 1993).

Of course, since the image in the denominator will vanish off actual reflector images, care has to be taken to avoid division by zero. Possible ideas of avoiding division by zero include the addition of a stabilization parameter to the denominator or masking the division so that it is carried out only at points where the denominator exceeds a certain threshold value.

NUMERICAL EXAMPLES

We have applied the above technique of velocity model building to the Marmousoft data (Billette et al., 2003). These data were constructed by Born modeling in a smoothed version of the Marmousi model. The true Marmousoft velocity model is depicted in Figure 1.

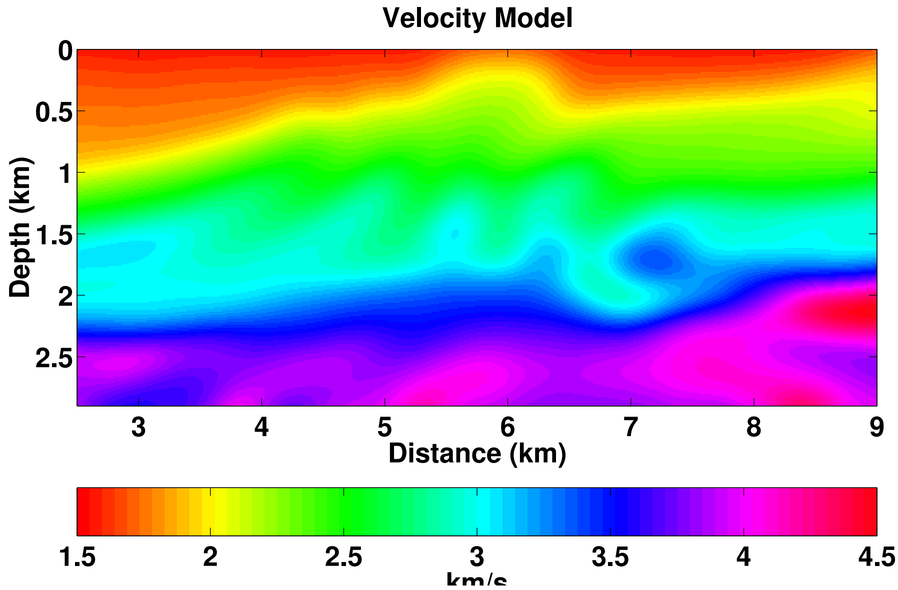

Figure 1: Marmousoft velocity model.

We carried out a multipath-summation time migration out using constant migration velocities between 1.4 km/s and 4.2 km/s in intervals of 25 m/s. This velocity sampling is sufficiently dense to satisfy condition 2 from the introduction. The resulting stacked migrated image is shown in Figure 2. We see that the multipath-summation approach produces a very nice image that exhibits the main structures of the Marmousi model, even though the central part of the image is not perfectly recovered. This is due to the intrinsic limitations of time migration rather than those of multipath summation.

Simultaneously, we carried out a second multipath-summation time migration using the same velocity values. It differed from the first one only by the use of the migration velocity as an additional weight factor in the stack. The resulting migrated image is shown in Figure 3. It looks quite similar to the unweighted stack result. As the only difference, we immediately note the increasing amplitudes with depth in comparison to Figure 2, indicating the increasing velocities that the amplitudes of Figure 3 carry. As indicated by the colorbar, the migrated amplitudes are in the range of seismic velocities.

The division of the images of Figures 2 and 3 results in a migration velocity model. Figure 4 shows the result when the division is stabilized by adding a fraction of the maximum amplitude to the denominator. We recognize that the overall trend of the velocity is nicely recovered, thus indicating that the velocity extraction by double multipath summation can actually work. However, the velocity model is rather unstable, with many image points where unreliable and wrong velocities have been extracted. These velocities already indicate the necessity to post-process this velocity model in order to extract only the meaningful velocities.

The best way to evaluate the quality of a time-migration velocity model is to actually use it for time migration. As shown in Figure 5, the velocity model obtained with the stabilized division (already with a rather small ϵ added) does not lead to an acceptable migrated image. Tests with different values for the stabilization parameter did not help to improve the image. The reason is that ungeologically low velocities the time migration cannot deal with are attributed to many locations in the model. These velocity values need to be eliminated by post-processing.

Since nongeological values must not be allowed in the final velocity model, an obvious idea is to already simply avoid division where the denominator is too small, and also discard velocities that are out of the range of velocities that were actually used for the multipath migration. Figure 6 shows the result of such a masked division, where zero is attributed to the velocity model wherever the denominator is too small to allow for a division or where unacceptable velocity values result from the division. This eliminates the incorrect velocity values but replaces them by zeroes, thus creating the need for velocity interpolation.

This can be clearly seen from the resulting time migration using this masked velocity model (see Fig-

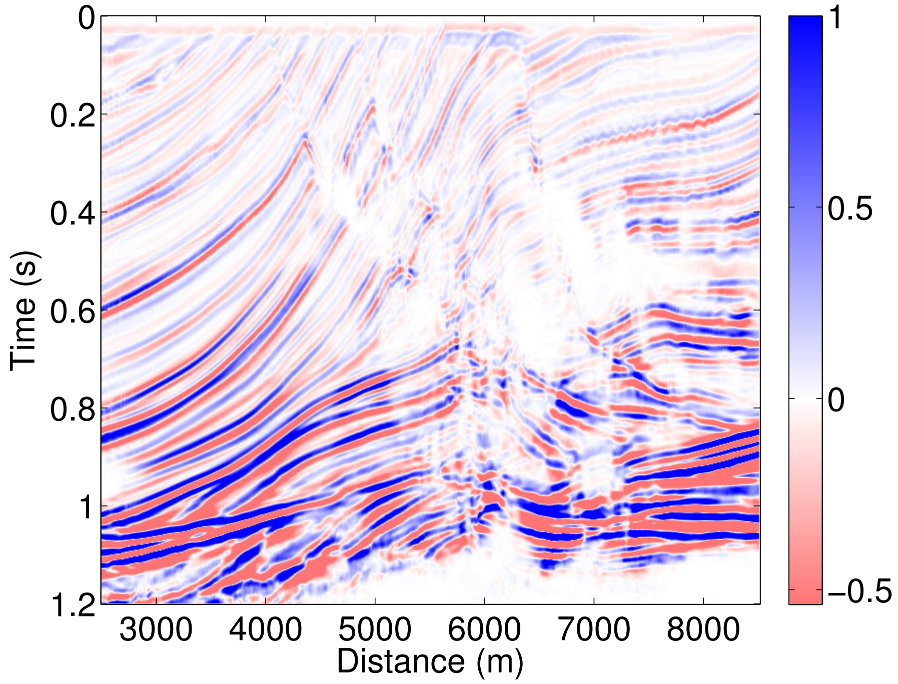

Figure 2: Result of multipath-summation time imaging.

ure 7). In this case, the migration was actually carried out only for those grid points where the velocity is different from zero We see that the migrated image nicely focuses the reflectors in the less complex areas, however creating some holes in the more complex parts.

A simple fill of the missing velocity values by the nearest nonzero neighbor leads to the velocity model shown in Figure 8. This velocity model no longer contains any zeros, but still is not smooth enough to be acceptable as a time migration velocity model. This can be confirmed from the resulting time-migrated image (Figure 9). While many of the holes in the image have been filled, leading to a more complete image, there is still room for further improvement.

Since a time-migration velocity model theoretically consist of rms velocities, it is supposed to be smooth. Rather than smoothing the model in Figure 8, we chose to directly smooth to the masked model of Figure 6, testing two kinds of smoothing techniques.

First, we applied moving-average smoothing using a window in which zero values of the velocity were ignored. It turned out that passing a smaller window several times yields a more reliable result than passing a larger window only once. Figure 10 depicts the resulting smooth velocity model after four passes of a smoothing window of 25 traces by 17 time samples. Note that the resulting velocity model closely resembles the model constructed with image-wave propagation in the image gather (Schleicher et al., 2008, see also this WIT Report).

The smoothed model considerably increases the image quality in the complex bottom and center parts (Figure 11). While the image is still not perfect in this region, this problem should be attributed to the general limitations of time migration in geologically complex areas rather than taking it as an indication of a poor velocity model.

Another way of obtaining a reasonably smooth time-migration velocity model from the masked velocities in Figure 6 is by B-splines interpolation. In this technique, B-splines coefficients on a regular grid are optimized by regularized least squares using all the available velocity information. The resulting velocity model for a moderate regularization is shown in Figure 12. The velocity model is rather similar to the one obtained with moving-average smoothing (Figure 10). The same applies to the time-migrated image (Figure 13). It is hard to spot significant differences between the two migrated images in Figures 11 and 13. Most of the slight differences that do exist occur in the center part of the model, where the geology is so

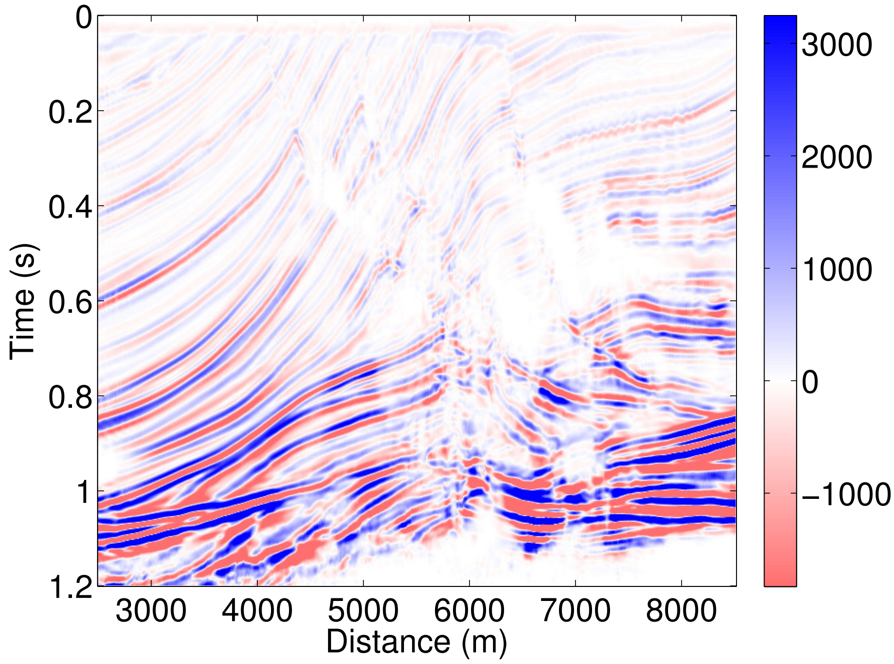

Figure 3: Result of multipath-summation time imaging with additional velocity weight.

complicated that time migration cannot realistically be expected to correctly position the reflectors.

Common-image gathers allow for a more detailed evaluation of the quality of the migration velocity model. Figure 14 shows six common-image gathers at positions 3000 m to 8000 m at every 1000 m in the moving-average smoothed model of Figure 10. We observe that these gathers are nicely flattened in the more regular parts of the model. The method only has difficulties to flatten the gathers in the central part of the model, where the geologic complexity is effectively prohibitive for any kind of time migration. For comparison, Figure 15 show the corresponding image gathers as obtained with the B-splines model of Figure 12. Even in the image gathers, it is very difficult to see differences between the two results.

CONCLUSIONS

The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocities. Those velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. Other CIGs cancel each other in the final stack. An exponential weight function using the event slopes in the CIGs helps to enhance the constructive interference and to reduce undesired events that might not be completely canceled by destructive interference.

Evaluation of the resulting path integral with Laplace's method demonstrated that the resulting image is proportional to the image that would be obtained with the correct velocity model. By executing the path-integral imaging a second time with a modified weight function including the migration velocity as an additional factor, an additional image is obtained the amplitudes of which are proportional to the stationary values of the migration velocity. Thus, these stationary velocities that produce the final image can then be extracted by a division of the two images. We have demonstrated with a numerical example that meaningful information about the migration velocity can be extracted from such a double path-integral migration.

Since multipath-summation imaging does not rely on any kind of interpretation, this technique allows for the fully automated construction of a first time-migrated image together with a first time-migration velocity model that can then be used as a starting model for subsequent velocity analysis tools like migration velocity analysis or tomographic methods.

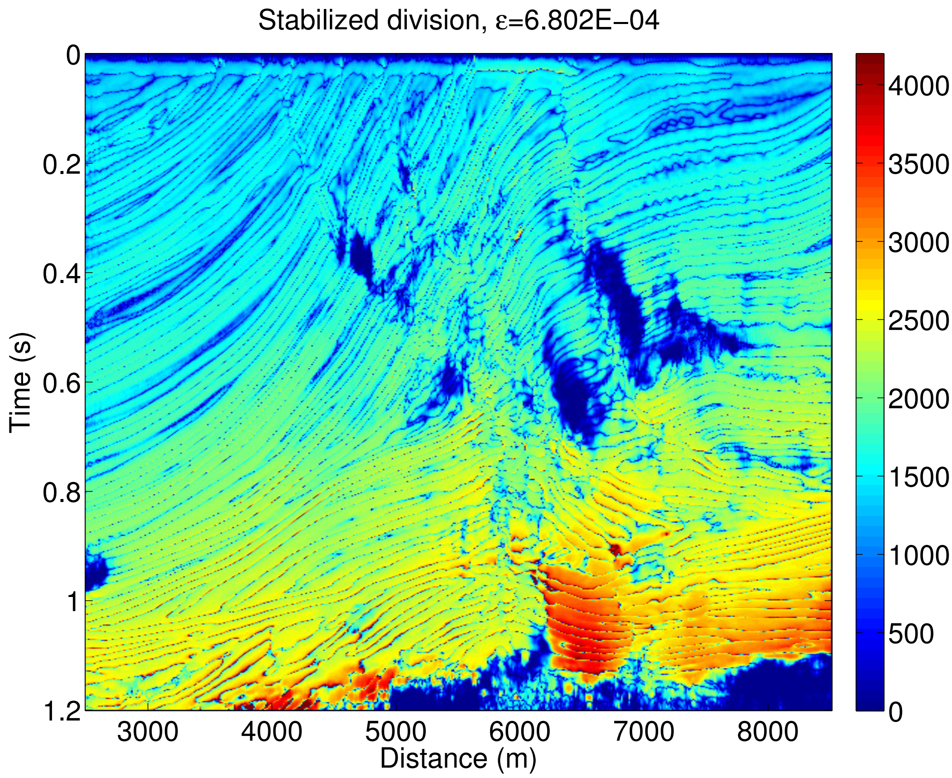

Figure 4: Velocities extracted by stabilized division.

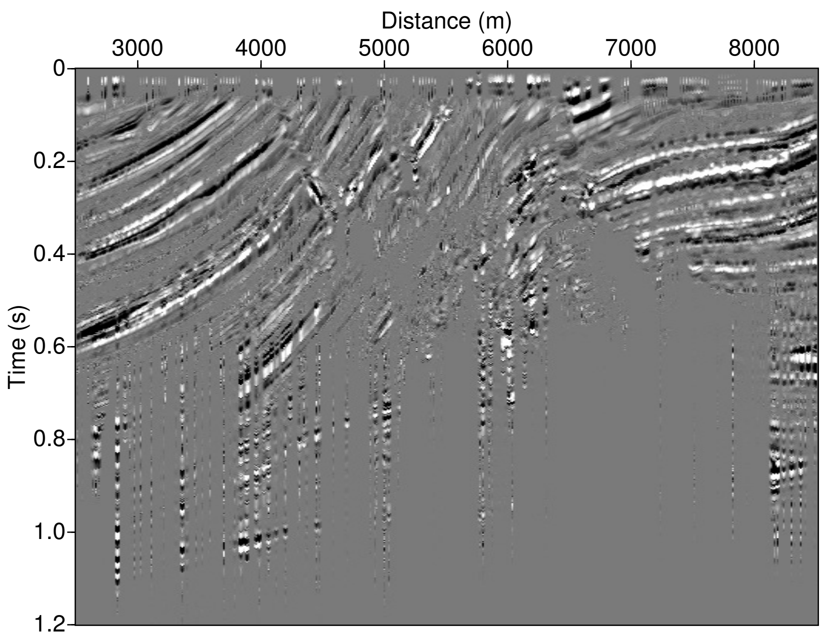

Figure 5: Time migration using velocities extracted by stabilized division.

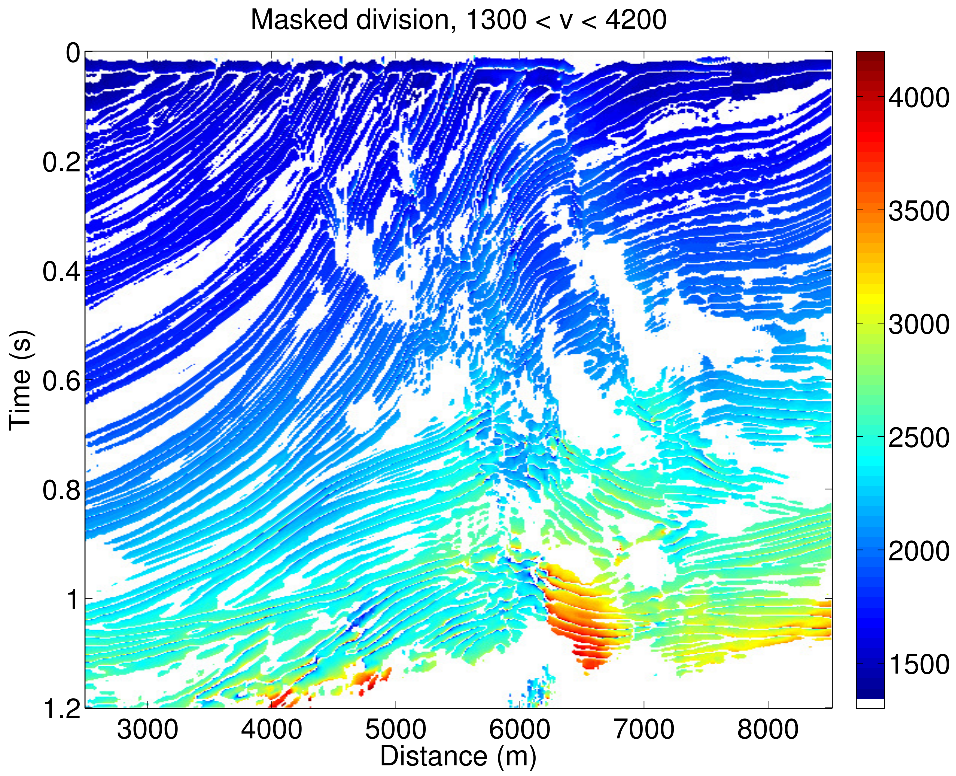

Figure 6: Velocities extracted by masked division.

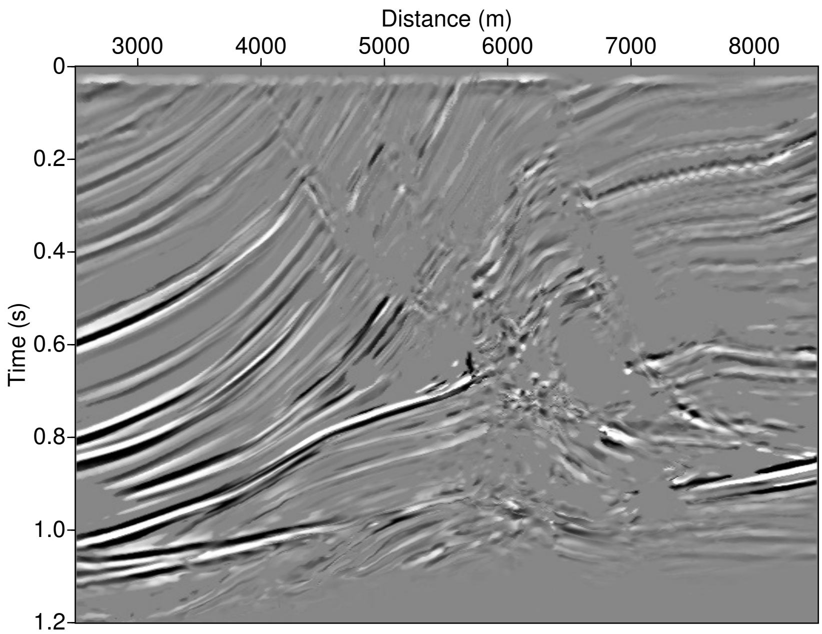

Figure 7: Time migration using velocities extracted by masked division.

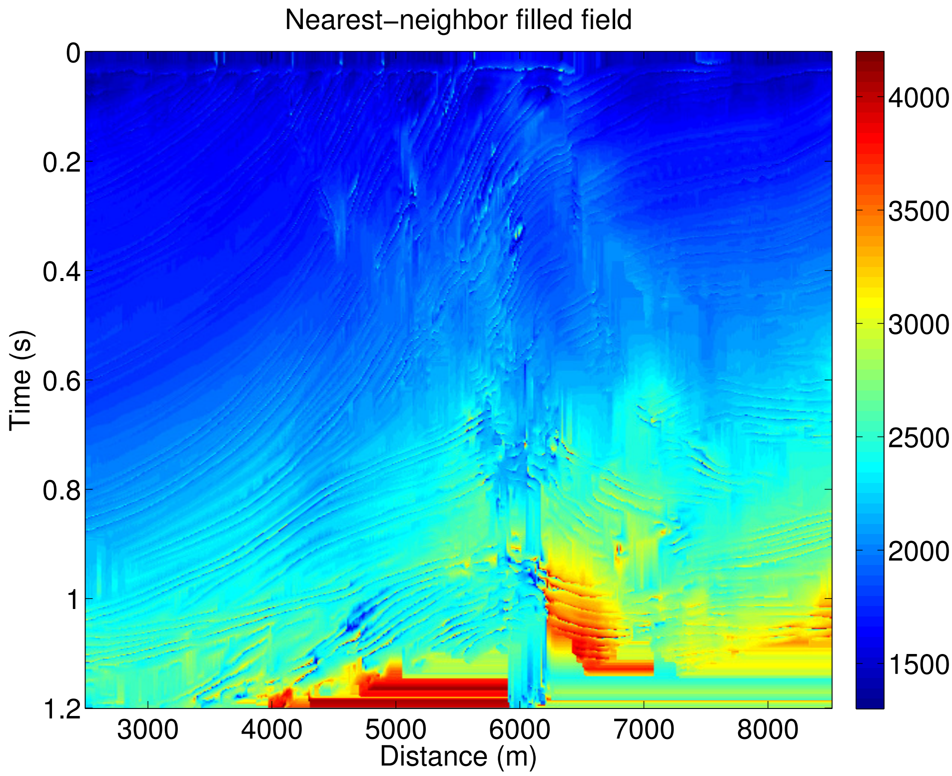

Figure 8: Velocities extracted by masked division plus nearest-neighbor filling.

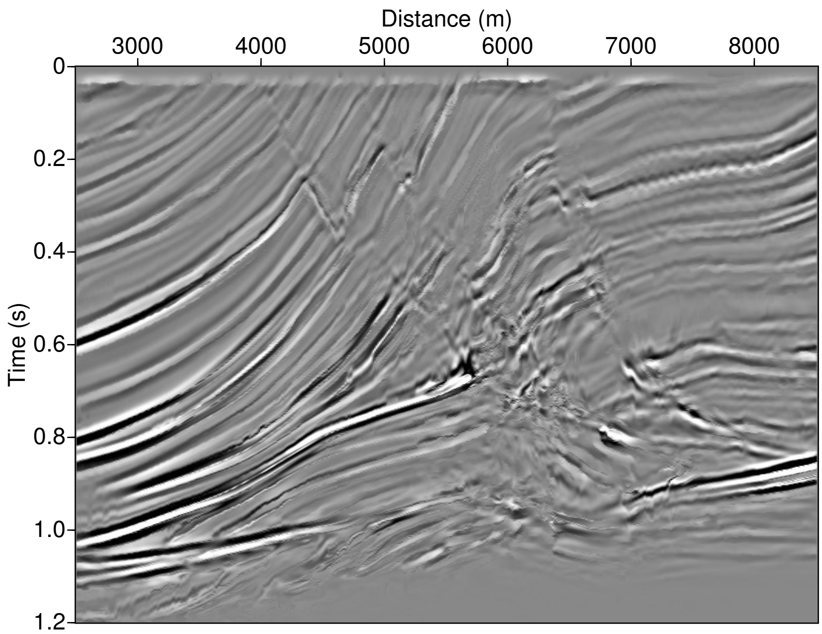

Figure 9: Time migration using velocities extracted by masked division plus nearest-neighbor filling.

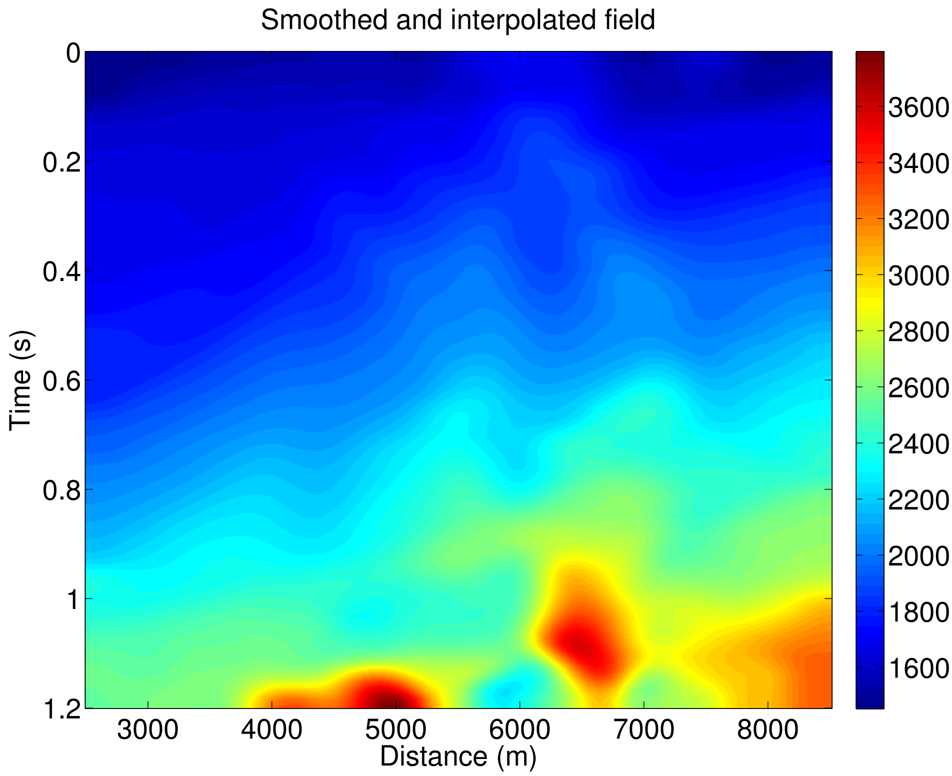

Figure 10: Velocities extracted by masked division plus moving-average smoothing.

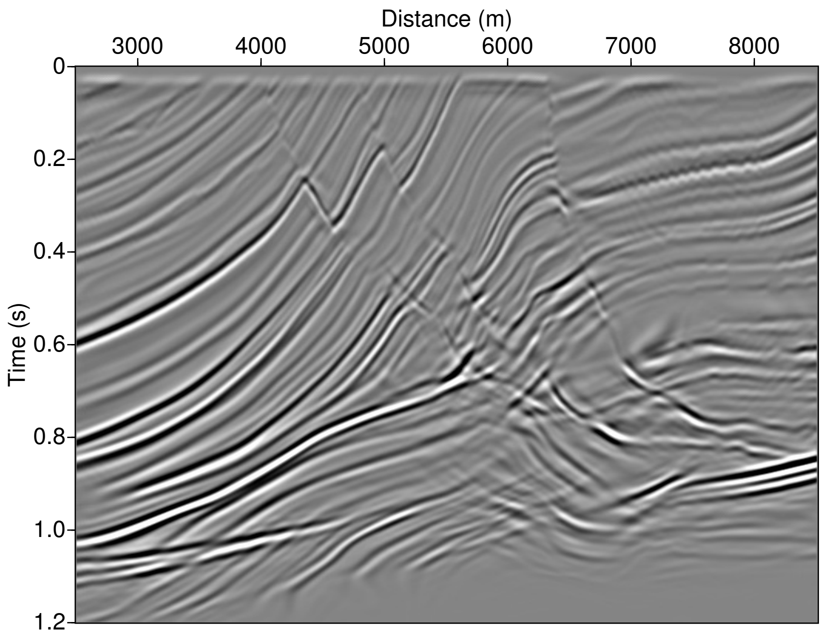

Figure 11: Time migration using velocities extracted by masked division plus moving-average smoothing.

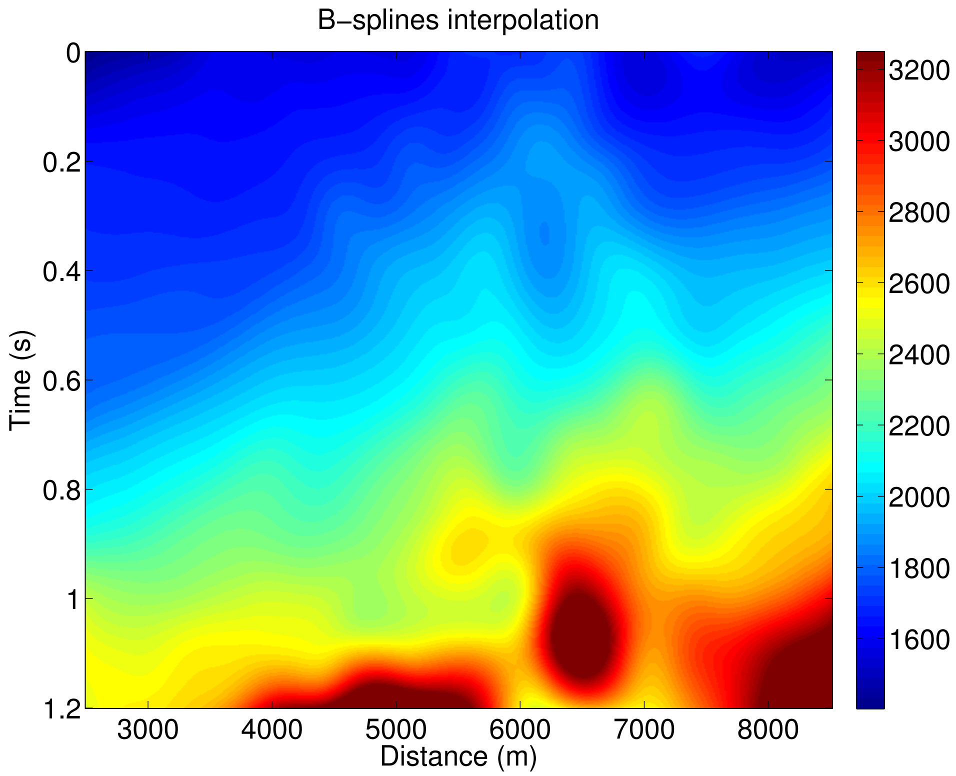

Figure 12: Velocities extracted by masked division plus B-splines smoothing.

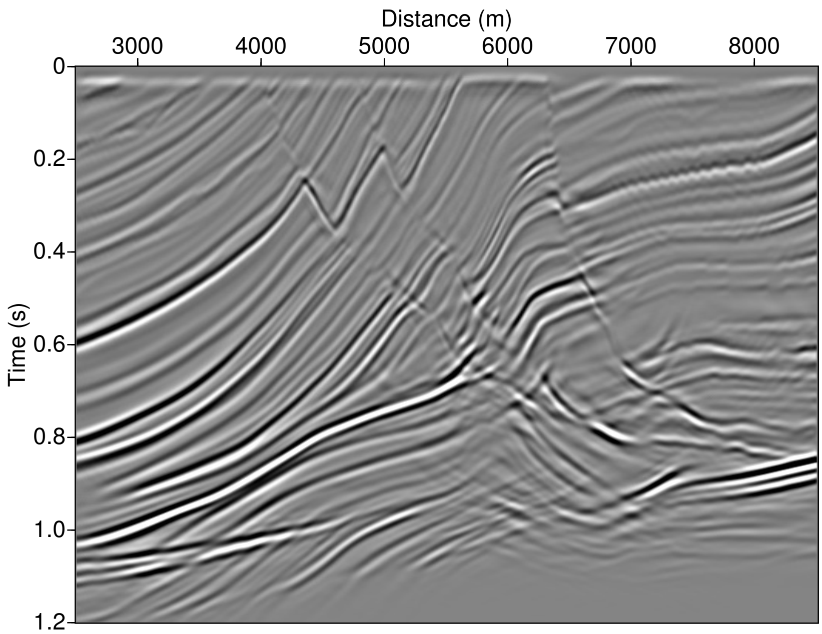

Figure 13: Time migration using velocities extracted by masked division plus B-splines smoothing.

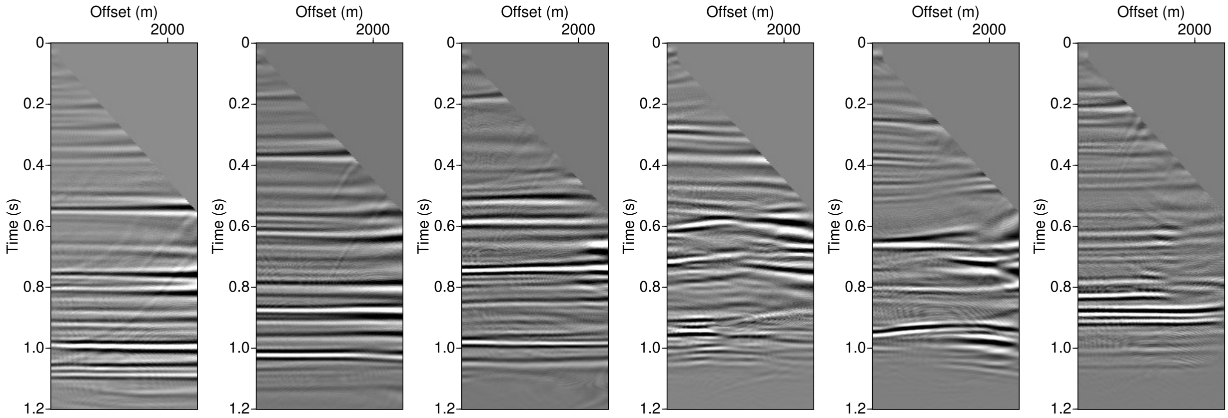

Figure 14: Common-image gathers from time migration using velocities extracted by masked division plus moving-average smoothing.

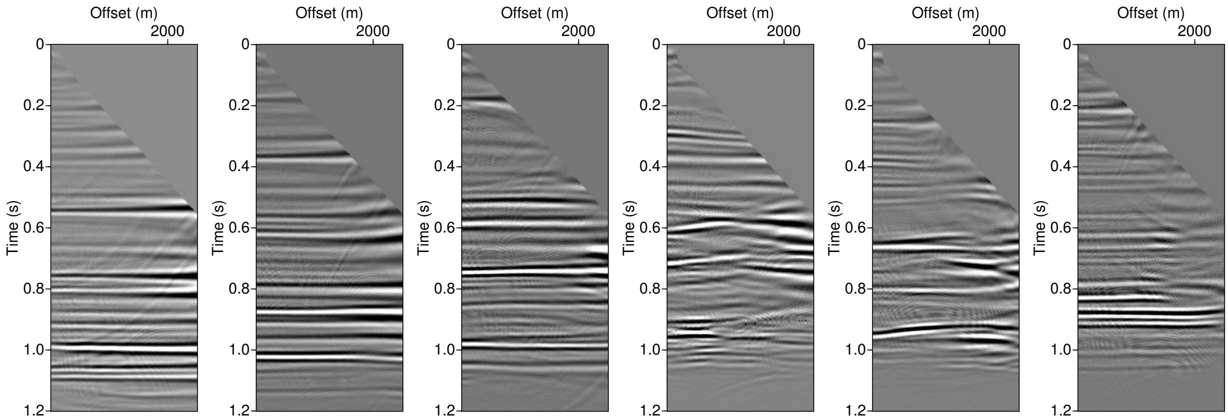

Figure 15: Common-image gathers from time migration using velocities extracted by masked division plus B-splines smoothing.

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