OPTIMIZATION OF COMMON REFLECTION SURFACE (CRS) ATTRIBUTES BASED ON A HYBRID METHOD

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ABSTRACT

The simultaneous estimation of Common Reflection Surface stack attributes considers a solution of a global non-linear minimization problem. We propose a hybrid method to solve this unconstrained optimization method. The approach comprises a conjugate direction method with its well known convergence properties and an iterative line search considering the strong Wolfe-Powell rule for the control of the step length. The use of the conjugate direction method leads to a highly efficient iterative search method to speed up the convergence rate while using Hessian is avoided. The iterative line search considering the strong Wolfe-Powell rule for the control of the step length prevents the premature convergence into local minima, without the need of computing the gradient. In the current version of the CRS code the Nelder-Mead optimization method is applied to estimate CRS stack attributes. This technique requires more iterations and is more time consuming than the hybrid method introduced here. Applications show that the method provides very good solutions particularly in the presence of several local minima. For the Sigsbee2A synthetic data the new optimization approach is 5 times faster in computational time compared to the Nelder-Mead optimization method.

INTRODUCTION

The common reflection surface (CRS) stack is a zero-offset simulation method in seismic processing (see, e.g., Müller, 1999). The 2-D CRS description of the reflection response is specified by three kinematic wavefield attributes, namely the emergence angle α of the normal ray, the radius of curvature R_{NIP} and R_N of the NIP and the normal wave, respectively. The CRS hyperbolic approximation is given by

$$t^2(x_m,h) = \left(t_0 + 2\frac{sin\alpha}{v_0}(x_m - x_0)\right)^2 + 2\frac{t_0 cos^2\alpha}{v_0}\left(\frac{(x_m-x_0)^2}{R_N} + \frac{h^2}{R_{NIP}}\right), \qquad (1)$$

where v_0 is the near-surface velocity. For each point (x_0, t_0) of the zero-offset volume, the CRS formula (1) allows to identify, collect and stack all events within the given aperture in the midpoint-offset domain. We can obtain a time image based on the specular reflection of normal incident rays by repeating this process (Mann et al., 2001; Cristini et al., 2002). In this paper, we propose a computational strategy to efficiently and accurately estimate the CRS attributes $\{\alpha, R_{NIP}, R_N\}$. Our strategy for the simultaneous estimation of the attributes is based on the coherence analysis and solving a global non-linear minimization problem for each point (x_0, t_0). The resulting conjugate direction algorithm, based on the Powell line search method, provides very good solutions even in the presence of several local minima without the need of computing gradients (Bonomi et al., 2009).

OPTIMIZATION PROBLEM

The CRS stack is implemented as a fully automatic procedure. This implies the computation of the kinematic attributes $\{\alpha, R_{NIP}, R_N\}$, which is also done in an automatic manner. The attributes are determined by means of the coherence analysis which essentially consists of maximizing the semblane criterion, given by:

$$S = \frac{1}{N} \frac{\sum_t \left(\sum_{i=1}^{N} f_{i,t(i)} \right)^2}{\sum_t \sum_{i=1}^{N} f_{i,t(i)}^2}, \tag{2}$$

where $f_{i,t(i)}$ is the amplitude on the i-th trace at traveltime $t(i)$ and N is the number of traces. The maximum coherence is achieved if there is a optimal fit between the CRS traveltime and the data.
Since for each new value of $\{\alpha, R_{NIP}, R_N\}$ the estimation of the semblance has a high computational cost in term of floating-point operations and data movement, the search process, if not correctly designed, might be very time consuming. This lead to the introduction of the pragmatic approach which subdivides the 3-D global search in the full pre-stack data volume into three 1-D searches in data sub-volumes. The number of traces in these sub-volumes is considerably lower and therefore the coherence does not have the S/N ratio it would displays in the full data volume for the global search. Furthermore, there is no guarantee that the resulting attributes may be wrong or linked to a local minimum.
Global optimization methods may overcome some of these deficiencies and problems. Since the derivatives of the semblance are not explicitly available, a gradient-based minimization scheme cannot be implemented. In the current implementation of the CRS method the Nelder-Mead approach is used for the optimization. This method does not need gradients and is therefore particularly suited to determine CRS attributes. An alternative and efficient way to keep good convergence properties without computing the gradients is to adopt a conjugate-direction method (Stewart, 1973), together with a reliable line search algorithm. We propose a strategy to simultaneously estimate common reflection surface (CRS) attributes using the conjugate direction approach based on Powell is search method. It allows to prevent the premature convergence being trapped into local minima. The idea is to steer the search using a strong Wolfe-Powell (SWP) rule.

Conjugate direction method

While solving linear systems with direct methods it is very difficult to adapt distributed memory systems for sparse matrix coefficients. Most of the sparse linear system solvers for distributed memory system use iterative methods. The conjugate direction method is a highly efficient iterative search technique because it allows to speed up the convergence rate of steepest descent while using Hessian matrix evaluation is avoided (Stewart, 1973). The iterations converge quadratically, starting from any initial guess and then trying to repeatedly improve the values. The old value is replaced by the new one with a special choice of direction determined by a line search. In every iteration a direction vector, d_i, and step length in the direction, α_i, are calculated to determine the next approximation of a new value in a line search method.

$$x_{i+1} = x_i + \alpha_i d_i. \tag{3}$$

Please note that in the second term of the equation no summation is considered, i.e., it is not a scalar product and i is a free index. This means the products need to be considered component wise. This applies to the following equations in this paper in a similar way.
Unfortunately, the search of a good solution minimizing a multimodal cost function has to deal with the possibility of a premature convergence to a local minimum. No optimization method based on a descent algorithm will prevent the trapping into local minima without allowing to escape movements along the opposite direction to the descent lines (Press et al., 2007). To determine a new point $(x_i + \alpha_i.d_i)$, an effective adaptive selection rule for the step length must be carefully implemented. The introduction of an uphill movement along each conjugate line towards regions of lower elevation greatly increases the capability of the line search algorithm to seek for solutions at lower cost. A good choice is the adoption of the strong Wolfe-Powell rule (Minoux, 1983). The idea is to steer the search using strong Wolfe-Powell rule to frame admissible solutions also along the counter-descent direction, i. e., to determine whether a

step length for a new point is significantly better than the current one.
Under these conditions, the fundamental property of these methods (Powell, 1964) is, that the result of the n-th iteration is exactly the minimum of $f(x)$ over the set d_i of conjugate directions in the attribute space. More precisely, if n nontrivial vectors d_i, $i = 1, 2, 3.....n$, are mutually conjugate, the exact minimum of $f(x)$ can be obtained by a sequence of n one-dimensional searches: starting at point x_0, the final result $x_{min} = x_n$ is extracted from:

$$f(x_{i+1}) = min \quad \{f(x_i + \alpha_i d_i)\}, \qquad i = 1, 2, 3...n. \tag{4}$$

The goal of the line search is to minimize $h(\alpha) = f(x + \alpha d)$ along the direction d varying the step length $\alpha_{LB} \le \alpha \le \alpha_{UB}$. For each coordinate (x_0, t_0) of the zero-offset volume, the minimization algorithm can be sketched as follows:

Powell Conjugate-direction algorithm

1. Initialiaties: set $x_0 = p_0$ and choose $d_1, d_2, ..., d_n$ linearly independent
2. FOR $1 \le i \le n$ DO
   Line search: find $\alpha_{LB} \le \alpha_i \le \alpha_{UB}$ minimizing $f(p_i + \alpha_i.d_i)$
   Define the new point: $p_{i+1} = p_i + \alpha_i.d_i$
3. Find an integer $1 \le k \le n$, so that $\delta = f(p_{k+1}) - f(p_k)$ is maximum
4. Compute: $f_3 = f(2p_n - p_0)$ and $f_1 = f(p_0)$, $f_2 = f(p_n)$
5. IF $f_3 < f_1$ AND $(f_1 - 2f_2 + f_3)(f_1 - f_2 - \delta)^2 < \frac{1}{2}\delta(f_1 - f_3)^2$ THEN
   Define the new direction : $d_k = p_n - p_0$
   Line search: find $\alpha_{LB} \le \alpha_k \le \alpha_{UB}$ minimizing $f(p_n + \alpha_k.d_k)$
   Define the new point: $x_{i+1} = x_i + \alpha_k.d_k$
   ELSE
   Keep all directions $d_1, d_2, ..., d_n$ for the next iteration and set $p_0 = p_n$
6. Repeat step 2 through 5 until convergence is achieved

NUMERICAL EXAMPLE

First we will test the method on an analytical example and compare it to the Nelder-Mead optimization procedure currently used as the optimization technique in the determination of CRS attributes. We are particularly interested in its behavior in the presence of a local minimum. For this purpose we consider the function $f(x_1, x_2) = \frac{1}{4}x_1 + 5x_1^2 + x_1^4 - 9x_1^2 x_2 + 3x_2^2 + 2x_2^4$. Figure 1 shows two different optimization methods with two different initial values. The first initial condition (Fig. 1a) was chosen close to the first local minimum (2.1482, 1.5875) and the second initial condition (Fig. 1b) was chosen close to the second local minimum (-0.0250, 0.0009). The Nelder-Mead method (Fig. 1c) and (Fig. 1e) gets trapped into local minima for both initial values and also needs considerably more iterations. The Powell conjugate-direction method (Fig. 1d) and (Fig. 1f) avoids the premature convergence to be trapped into local minima and gets fairly close to the global minimum at (-2.1817, 1.6069). In the conjugate-direction approach the number of iterations corresponds to the number of variables of the function to optimize, i.e., 2 for the analytical example above, 3 for the 2-D ZO CRS operator and accordingly for the 3-D and offset operators.

In the next section we apply the Powell conjugate-direction optimization procedure to a complex synthetic example and compare the results to our current implementation of the CRS attributes determination utilizing the Nelder-Mead procedure. We test the optimization procedures on the Sigsbee 2A synthetic data. Sigsbee 2A is a constant density acoustic synthetic dataset released in 2001 by the Subsalt Multiples Attenuation and Reduction Technologies (SMAART JV) consortium. It models the geologic setting found in the Sigbee escarpment in the deep water Gulf of Mexico. Figure 2 shows the resulting ZO stacked sections. The ZO CMP stack section (Fig. 2a) displays well known features when comparing CRS (Fig. 2b) and CMP stacked sections. The CRS stack in Fig. 2b was obtained using the pragmatic approach without global optimization. Here the 3-D search is subdivided into three subsequent 1-D searches. Compared to the CMP stack in Fig. 2a it shows a better S/N ratio, improved continuity of reflections and also a better image for conflicting dips. Figure 3 shows CRS stacks with two different global optimization procedures where the initial conditions of the search were chosen from the results of the pragmatic approach. The

Nelder-Mead optimized CRS stack (Fig. 2a), and the Powell conjugate-direction optimized CRS stack (Fig. 2b). Both images are very similar and display less noise and scatter compared to the CRS image obtained with the pragmatic approach and no global optimization in Fig. 2b. The conjugate-direction approach, however, is 5 times faster in computational time since it requires less iterations.

Figure 4 to 6 display results of CRS attributes and coherence for the optimization methods using the Nelder-Mead (NM) approach and the Powell conjugate-direction (CD) approach. Figure 4 and 5 shows resulting sections of CRS stacking parameters α and R_{NIP}. The angle of emergence and curvature of NIP wave section for Nelder-Mead optimization method is displayed in Fig. 4a and 5a. The angle of emergence and curvature of NIP wave section for the Powell conjugate-direction optimization method is shown in Fig 4b and 5b. The sections are similar and no specific differences can be observed. The coherence section of CD in Fig. 6b displays a bit more scatter than the coherence of the NM section (Fig. 6a). As already mentioned the stacks of both methods look similar and it does not appear that the difference in coherence has a major effect on the attributes. This will be further investigated by evaluating the results of other procedures in the CRS workflow like, e.g., tomography, diffraction processing, or multiple suppression. Currently we would favor CD against NM because of the substantial computational advantage. This advantage of the conjugate-direction approach will be even more exposed when going to multi-parameter stacking formulas with more than 3 attributes, like offset CRS, converted waves or 3-D applications.

CONCLUSIONS AND OUTLOOK

We have proposed an alternative strategy for the optimization of CRS attributes. The method is based on a hybrid optimization, which comprises the conjugate direction approach based on a Powell search method. A specific feature of the conjugate-direction approach is its ability to avoid being trapped in local minima. Moreover, the Powell conjugate-direction method has a substantial computational advantage against the Nelder-Mead method currently used in the CRS attribute search. A factor of 5 in computing time was observed for the global search of the parameters in the 2-D CRS stack using the conjugate direction approach. For multi-parameter stacking approaches with more than 3 attributes (offset formulas, converted waves, 3-D) this advantage will be even more obvious since considerably less evaluations of the semblance coefficient are required where most of the CPU time is used in the CRS method. The quality of the attributes are the key for a successful application of the CRS workflow. Whereas we do not see specific differences in the stack or attribute sections for the conjugate-direction or Nelder-Mead approach, we will investigate the performance of the attributes in other processing steps, like, e.g., tomography, diffraction processing, or multiple suppression. In the numerical examples of this paper initial values in the global search were taken from the results of the pragmatic approach. In the future we want to completely avoid the pragmatic approach and move to global optimization right from the start. We expect that this strategy will further improve the quality of the attributes which should positively influence the results of the CRS workflow.

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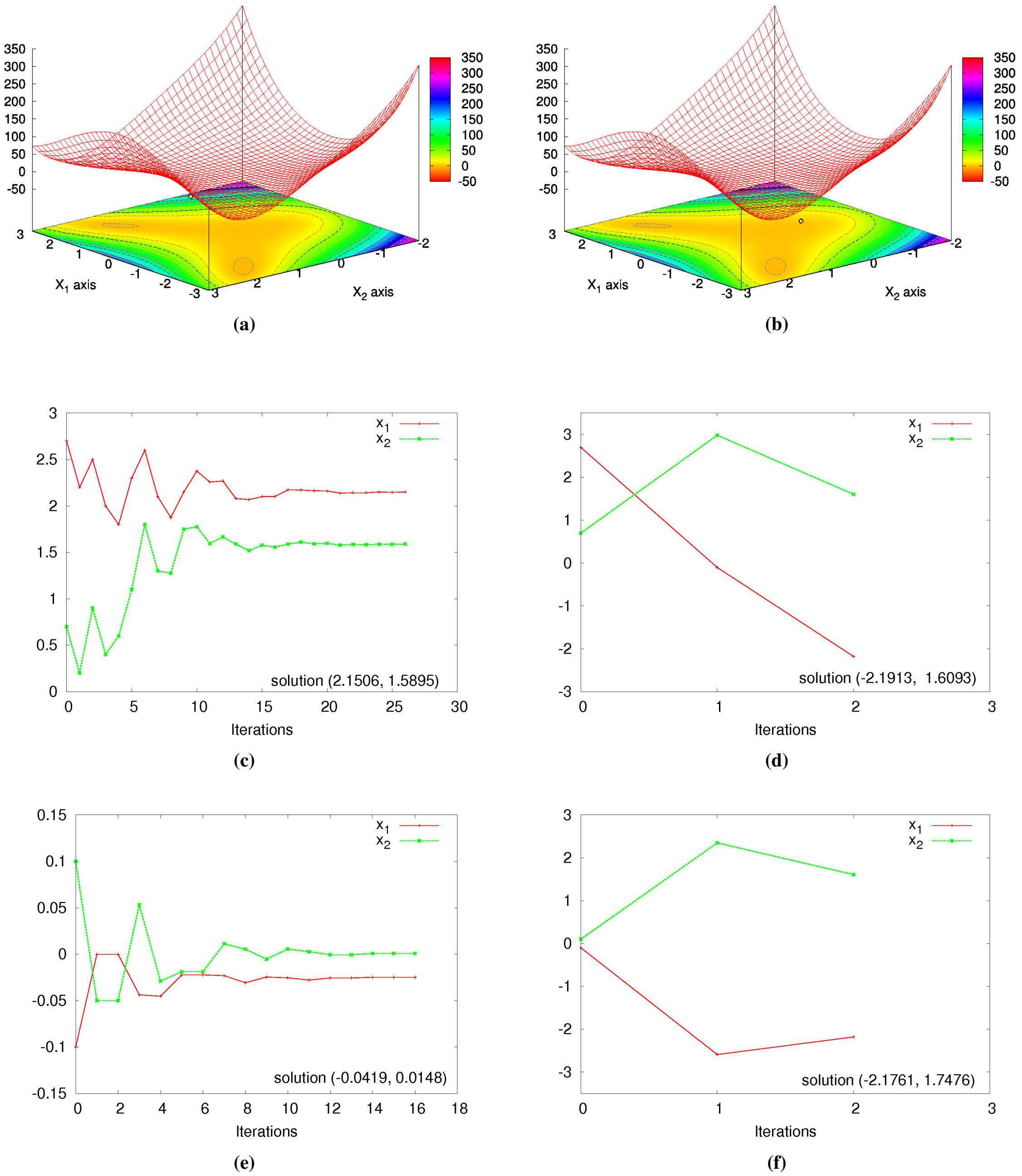
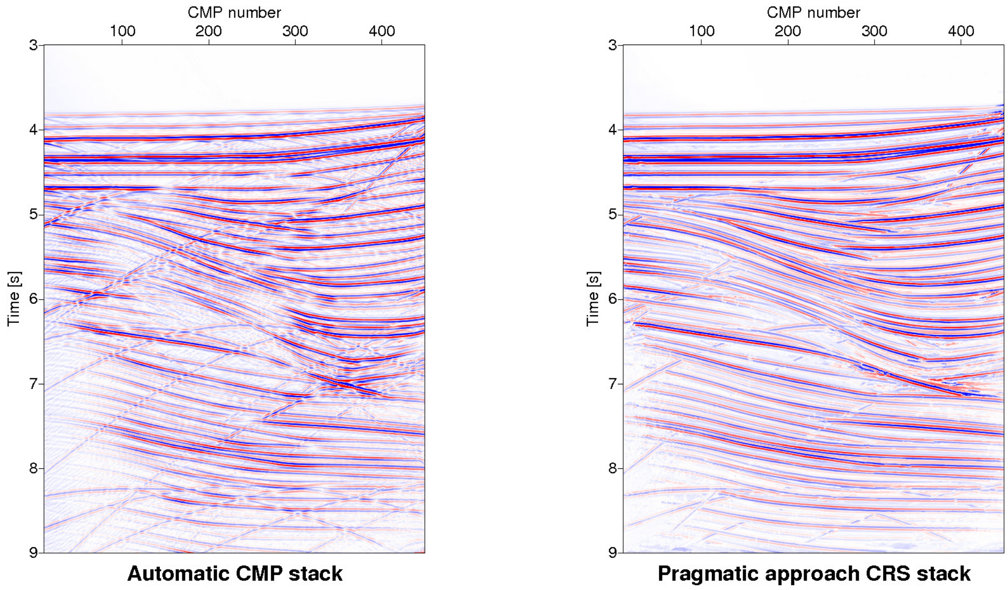
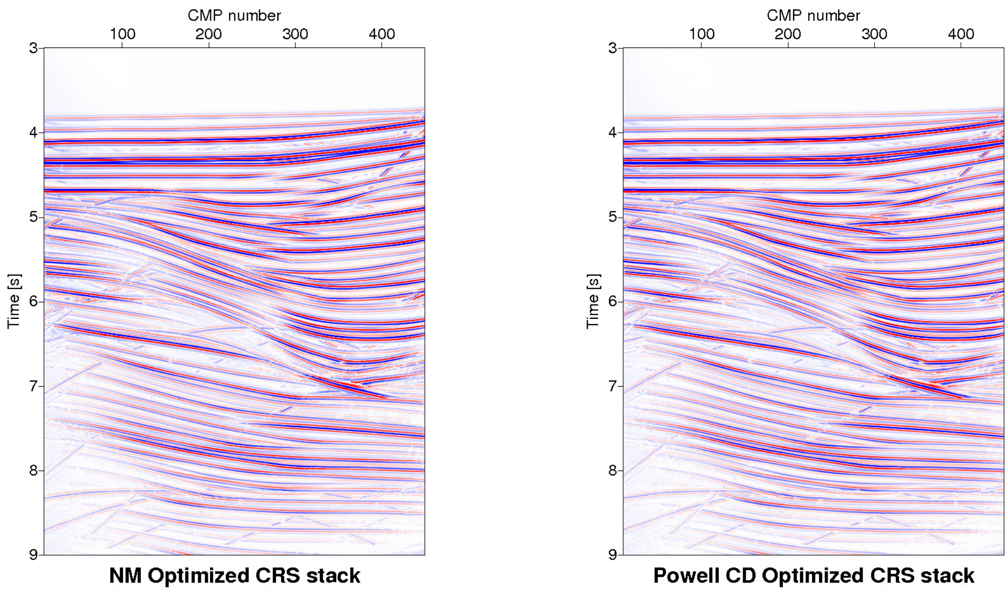


Figure 1: Two different optimization method with two different initial values: the first initial condition was chosen close to the first local minimum (2.1482, 1.5875) (a) and the second initial condition were chosen close to the second local minimum (-0.0250, 0.0009) (b). The Nelder-Mead method (c) and (e) get trapped in local minima. The Powell conjugate-direction method (d) and (f) avoids the premature convergence into local minima and gets close to the global minimum at (-2.1817, 1.6069).



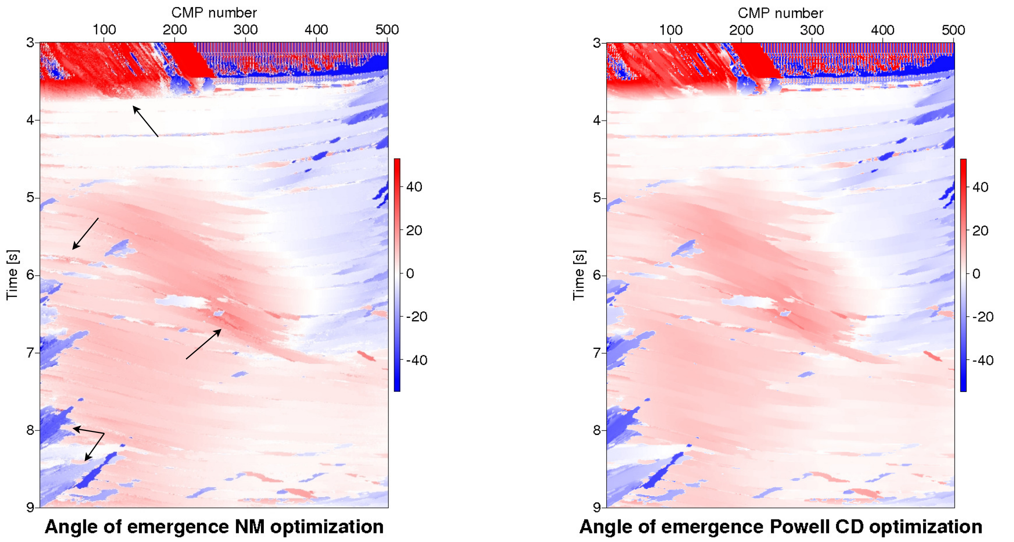
(a) **(b)**

Figure 2: Zero-offset stacked sections: comparison between the conventional CMP stack and pragmatic approach CRS stack. The conventional CMP stack (a) has lower quality in conflicting dip areas. In the pragmatic approach CRS stack (b), reflections are more continuous and appear clearer, diffractions are attenuated.



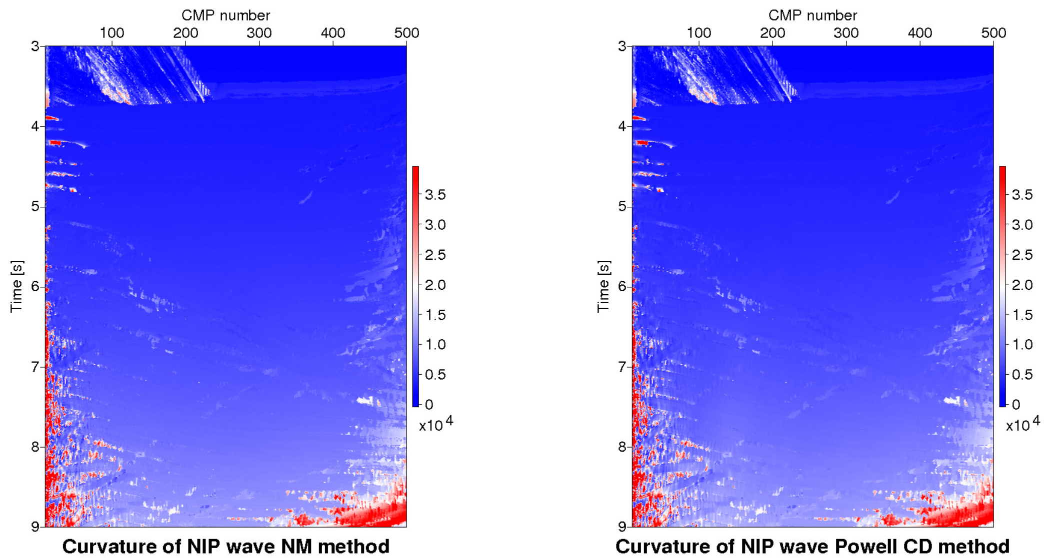
(a) **(b)**

Figure 3: CRS stacked sections: comparison between the Nelder-Mead optimization method and Powell conjugate-direction optimization method with the same initial values. The Powell conjugate-direction optimization method (b) is 5 times faster in computational time compared to the Nelder-Mead optimization method (a).



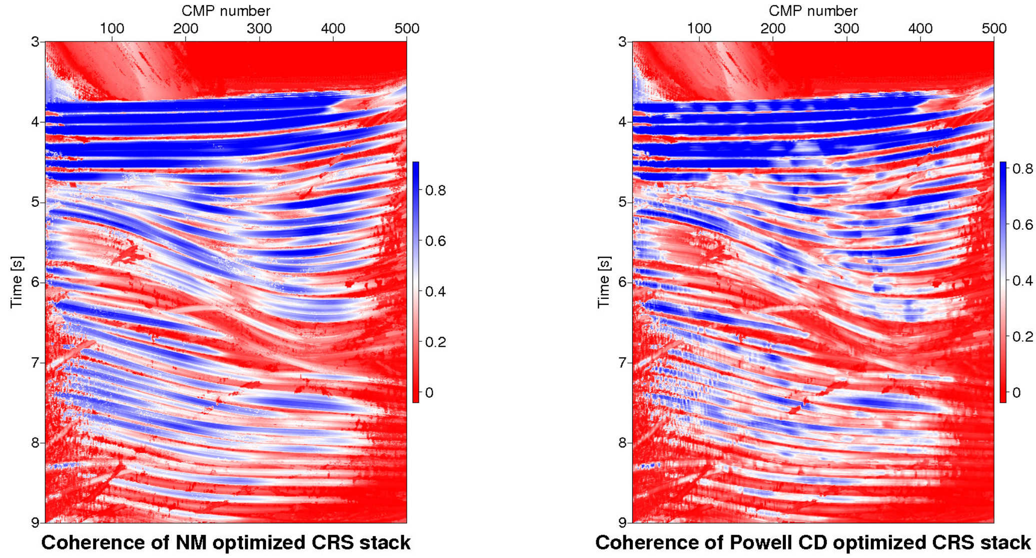
(a) (b)

Figure 4: Results of the automatic CRS parameter searches: α. Angle of emergence section for Nelder-Mead optimization method (a) and angle of emergence section for Powell conjugate-direction optimization method (b).



(a) (b)

Figure 5: Results of the automatic CRS parameter searches: R_{NIP}. Curvature of NIP wave section for Nelder-Mead optimization method (a) and curvature of NIP wave section for Powell conjugate-direction optimization method (b).



(a) (b)

Figure 6: Coherence section: comparison between the Nelder-Mead optimization method (a) and Powell conjugate-direction optimization method (b).

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