ON THE EFFECTS OF GEOMETRICAL SPREADING CORRECTIONS FOR A 2D FULL WAVEFORM INVERSION OF RECORDED SHALLOW SEISMIC SURFACE WAVES

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ABSTRACT

We aim for an inversion of recorded shallow seismic surface waves by a 2D elastic full waveform inversion approach. This approach has the potential to provide a high resolution subsurface model of the shear wave velocity. The wavefields of field data have a 3D geometrical spreading. However, a 2D synthetic forward modeling code produces wavefields with 2D spreading. Therefore a correction of the different geometrical spreading is required. The spreading correction consists of an amplitude-only correction and a phase-transformation. In the first part we consider an amplitude-only correction and present synthetic inversion tests assuming line source seismograms but also amplitude corrected point source seismograms as observation. It turns out that the final models with line source wavefields are more reliable than with point source wavefields. Therefore we recommend to consider not only an amplitude-only correction but also a 3D/2D phase-transformation. In the second part we introduce two 3D/2D transformations known from literature for body waves and apply them to surface waves. Furthermore we consider a simulation of a line source during shallow seismic measurements. We demonstrate the adjustment for each transformation technique and show that they all perform quite accurate.

INTRODUCTION

The inversion of shallow seismic surface waves is very attractive for geotechnical investigations. Surface waves which are easily excited by a hammer blow have a high sensitivity to the shear wave velocity in the first 10 to 15 m of the subsurface. A hammer blow as a source dominantly excites surface waves and thus the signal to noise ratio of surface waves is very high compared to body waves. With surface waves it is possible to investigate sites with low-velocity zones which cannot be done with refracted body waves. There are established methods to invert surface waves (e. g. inversion of dispersion curves or wavefield spectra (Forbriger, 2003)) but all these methods assume 1D subsurface structures. This assumption is not satisfied in some applications of practical relevance. To overcome this limitation we want to apply an elastic full waveform inversion (FWI) to shallow seismic surface waves. The application of a 3D FWI to surface waves unfortunately is still difficult due to excessive requirements for computational resources. Therefore we apply a 2D inversion. The inversion code was developed by Köhn (2011). It is based on the adjoint method and the inversion is done in the time domain. The forward modeling is done with the Finite Difference method according to Bohlen (1998) and Bohlen (2002). First successful applications of a FWI of surface waves show the high potential of this method (Romdhane et al., 2011).

With the objective to realize a 2D full waveform inversion of field data, we have to deal with the different spreading of point and line sources. Observed data are commonly gained with a point source such as a hammer blow or an explosion which produce a wavefield with 3D spreading. However, a 2D inversion-code uses implicitly a line source and produces a wavefield with 2D spreading. Due to unequal decaying in amplitudes and a certain shift in phase point source and line source wavefields are different. These residuals could result in model artifacts during a full waveform inversion. To investigate the importance of this we realize inversions for synthetic line source seismograms as well as for synthetic point source seismograms without any 3D/2D phase-transformation.

The report is organized as follows. First we introduce an amplitude-only transformation that allows us to correct the point source wavefields to the amplitude spreading of a line source wavefield. As we are using the L2 norm this is a prerequisite for the full waveform inversion of point source seismograms. Afterwards we compare the inversion results obtained by a full waveform inversion of line source seismograms and amplitude corrected point source seismograms. This comparison is shown for a 1D and a 2D subsurface structure. From our results we conclude that applying only an amplitude correction to the point source seismograms is not enough to obtain reliable inversion results with a 2D FWI. Moreover we also have to correct for the phase differences between point and line source seismograms. In the second part of the report we therefore introduce some phase-transformation techniques and compare them for surface waves.

FIRST INVERSION RESULTS

Before we present and discuss our inversion results, we introduce briefly our inversion parameters and strategies during a FWI. The misfit between the observed and synthetic data is calculated with the L2 norm. Thereby the true amplitudes are taken into account and because of geometrical spreading near offset traces are more weighted than far offset traces. We use the information of the vertical and the horizontal components. The model parameters are the shear wave velocity v_s, the P-wave velocity v_p and the density ρ. We present only the v_s-models due to better resolution caused by the sensitivity of surface waves to the v_s-velocity. Furthermore we use a multistage approach with frequency filtering to reduce the nonlinearity of the misfit function. Starting at low frequencies builds up a smooth subsurface model and prevents the FWI to end up in a local minimum. We start at a low frequency of 10 Hz and increase the range of frequencies step by step up to 100 Hz which corresponds to the full bandwidth of the source signal. However, with point source seismograms this approach fails because it seems that the inversion ends up in a local minimum of the misfit function even so. The differences between seismograms of a point and a line source are in the nearfield stronger and for low frequencies the nearfield term dominates almost in the entire profile length. Therefore we don't use frequency filtering in the inversions of point source seismograms at the moment. For the source time function $s(t)$ we used the first pulse of a $\sin^3(t)$ which is defined as

$$s(t) = \begin{cases} 0 & \text{for } t \leq 0, \\ F_0 \sin^3\left(\frac{\pi t}{T_d}\right) & \text{for } 0 < t < T_d, \\ 0 & \text{for } t \geq T_d \end{cases} \quad (1)$$

with the duration of the source signal T_d=32 ms and the scalar force F_0=1 N.

Amplitude-only transformation

Seismograms excited by a point and a line source have different geometrical spreading. Due to the misfit calculation with the L2 norm we have to do an amplitude transformation to realize an inversion of point source seismograms with a 2D-FWI-code. Because we don't know the relation of the amplitude-decaying between a point and a line source for surface waves, we assume to find an offset dependent factor y, which has the form of

$$y(r) = A \cdot r^x \quad (2)$$

with offset r. The determination of the factor A and the exponent x is done with a least squares inversion by minimization of the misfit of rms-amplitudes and works really robust. In Figure 1 seismograms of a point and a line source are displayed. The amplitude-only transformation is applied to the point source seismograms. Therefore it is possible to compare true amplitudes. In this plot the seismograms are not trace normalized but both are scaled by an offset dependent factor $(r/1\,\text{m})^{0.4}$ due to a better visualization.

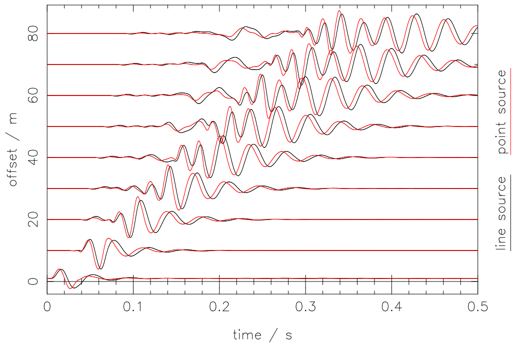


Figure 1: Comparison of a line source and a point source wavefield. The amplitudes of the point source seismogram are corrected by the amplitude-only transformation (no phase-transformation is used). The seismograms are not trace normalized but both are scaled by an offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.4}$.

With this approach it is possible to fit the amplitudes between a point and a line source. This is the first significant step to realize an inversion of point source seismograms with a 2D-FWI-code. In the following this amplitude correction is called amplitude-only transformation and is applied on every point source seismogram. Nevertheless, no phase-transformation is used so far.

Example for a 1D structure

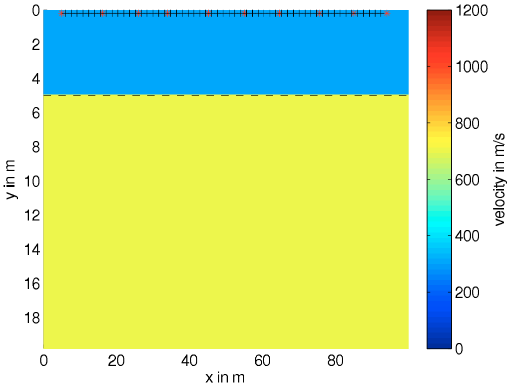
In a first step we start our inversion tests with line source and point source wavefields for a layered medium. This is in some way easier to handle and the residuals between point and line source wavefields are more linear. The true model is a layer over halfspace (Figure 2(a)) with the parameters given in Table 1.

	v_p in m/s	v_s in m/s	density in kg/m^3	Q_p	Q_s
layer (5 m)	500	300	1800	inf	inf
halfspace	1200	700	2000	inf	inf

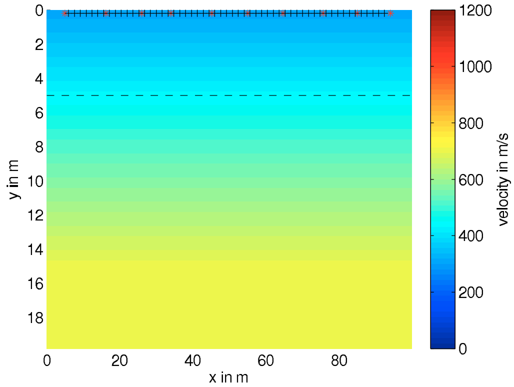
Table 1: Material parameters for a layer over halfspace.

For the starting model we used a linear gradient shown in Figure 2(b). We applied 10 vertical point sources like a hammer blow and recorded the wavefields with 88 equidistant receivers (horizontal and vertical component). First we test the FWI assuming line source seismograms as observation. In Figure 3(a) the wavefields of the true and starting model are plotted and we can clearly see that there are strong residuals between them. The inversion result with a line source wavefield is shown for the subsurface model in Figure 2(c) and the comparison of the wavefields between the true and the final model is plotted in Figure 3(b). The result of the inversion assuming a perfect line source seismogram as observation for a 1D structure is very promising. The depth of the layer is quite clearly carved out, just the velocity of the halfspace is not totally correct.

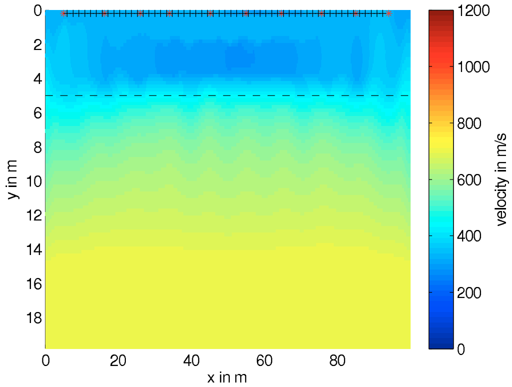
In the next step we test how accurate we can reproduce this inversion result with an amplitude corrected point source wavefield without a 3D to 2D phase-transformation. For the true and the starting model we have used the same models as for the inversion with line source wavefields (see Figures 2(a) and 2(b)). In Figure 4 the final subsurface model is plotted as well as in Figure 5 the comparison between the true, starting and final seismograms. The inversion result is quite surprising. On the one hand the layer is almost carved out (see Figure 4), but we can clearly see that the surface of the layer is not flat and that the velocity of the halfspace is not well estimated. On the other hand the final seismogram fits the true seismogram only in the first traces, with larger offsets we can notice a phase shift which is probably caused by the phase shift between a point and a line source and by uncertainties/artifacts in the subsurface model. It seems that the inversion-code could not explain this phase shift with a change or even a big artifact in the model. This



(a) v_s true model

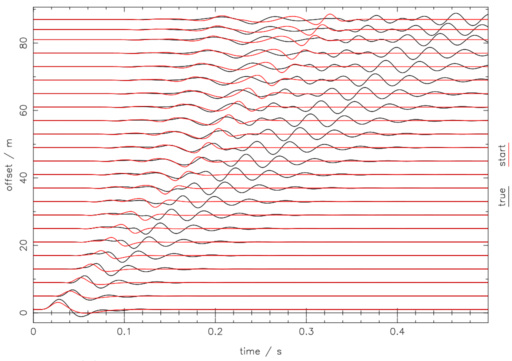


(b) v_s starting model

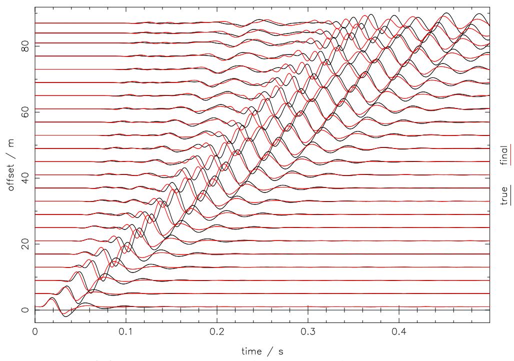


(c) v_s final model

Figure 2: a) true, b) starting and c) final model for layer over halfspace assuming a line source seismogram. Sources and receivers are denoted by * and +, respectively. The dashed line indicates the depth of the layer at 5 m.



(a) seismograms of true and starting model



(b) seismograms of true and final model

Figure 3: Comparison of the line source wavefields (vertical component) for shot at x=6 m between the true, starting and final model; not all traces are shown. The seismograms are not trace normalized but both are scaled by an offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.25}$.

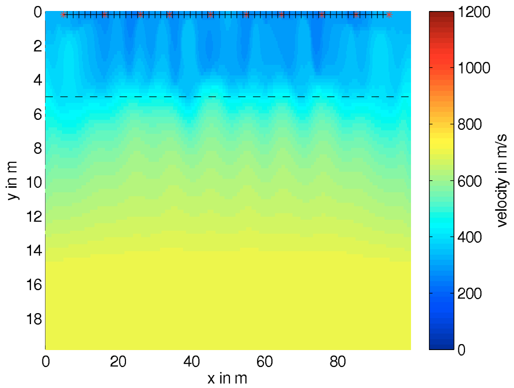
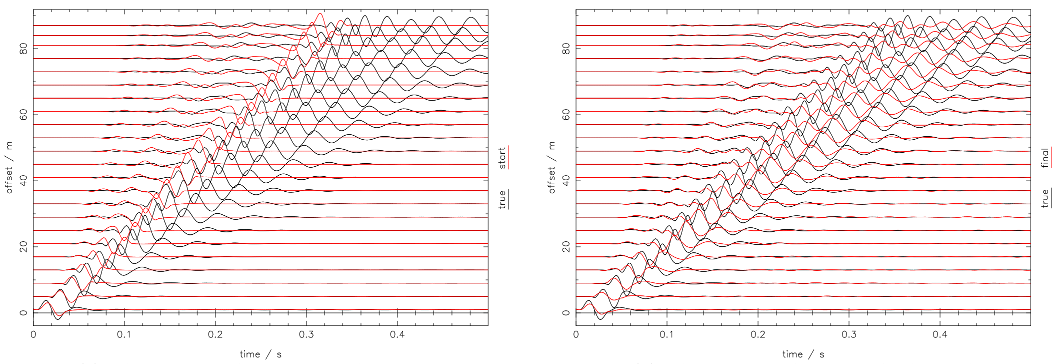


Figure 4: Final model for layer over halfspace assuming a point source seismogram as observation with amplitude-only correction (no phase-transformation is used). Sources and receivers are denoted by * and +, respectively. The dashed line indicates the depth of the layer at 5 m. 2D structures are observable within the layer.



(a) seismograms of true and starting model

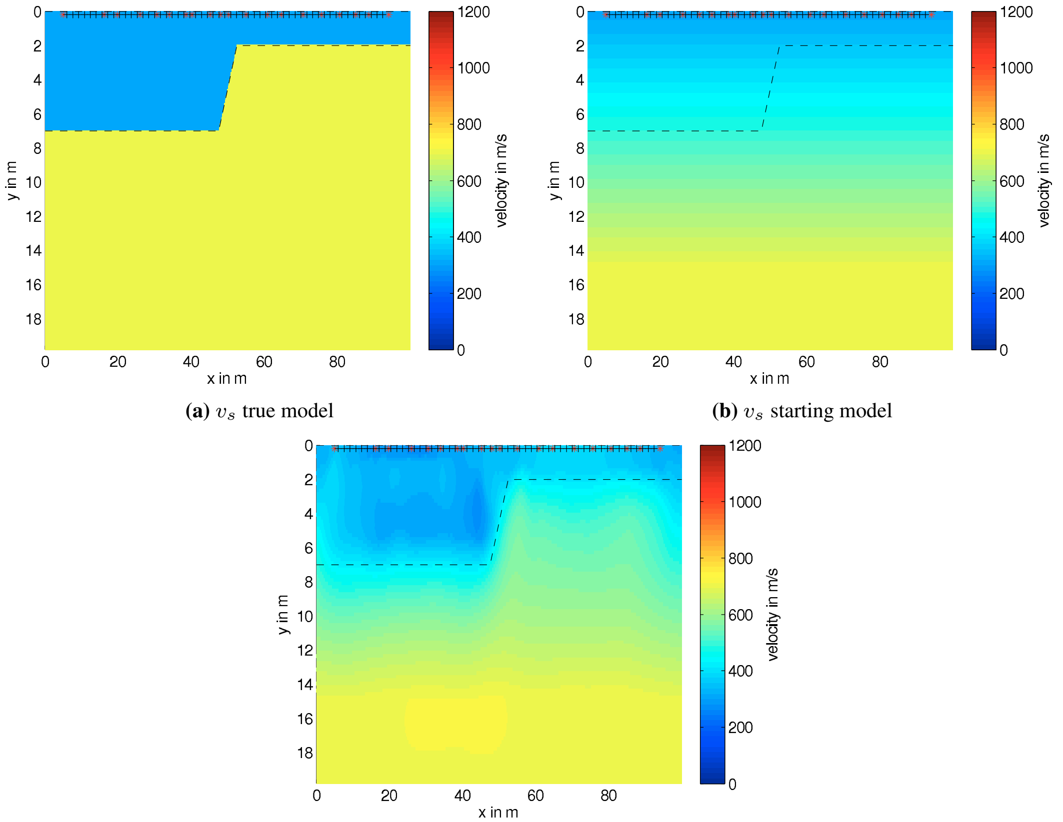
(b) seismograms of true and final model

Figure 5: Comparison of the point source wavefields (vertical component) for shot x=6 m between the true, starting and final model; not all traces are shown. The amplitudes of the point source seismogram are corrected by the amplitude-only transformation (no phase-transformation is used). The seismograms are not trace normalized but both are scaled by an offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.25}$.

means that for this 1D structure an inversion with point source wavefields does not completely fail, but there are more 2D structures observable than in the final model with line source seismograms. This could be also due to the missing frequency filtering during a FWI with point source seismograms.

Example for a 2D structure

In the next step of our inversion tests we extend our true model to a 2D structure. We choose a layer, which has a step, over a homogeneous halfspace (see Figure 6(a)). This corresponds to a field dataset we have recently acquired. First we assume line source seismograms as observation. For the starting model we use again a linear gradient (1D) which is plotted in Figure 6(b). The number of shots is similar to the real dataset with 20 vertical point sources. Receivers (number and positions) are the same as for the 1D structure. The inversion result is shown in Figure 6(c). In Figure 7 the comparison of the true, starting and final seismograms is plotted. The inversion result for a 2D structure with line source wavefields is very promising. The contour of the layer is fitted well, but the velocity of the halfspace is not yet reproduced. In Figure 7(a) and 7(b) we can clearly see that the misfit between the seismograms decreased and that the final seismogram fits the true seismogram. Also the backpropagated waves of the step are reproduced (offset: 0-40 m, time: 0.25-0.4 s, Figure7(b)).



(a) v_s true model

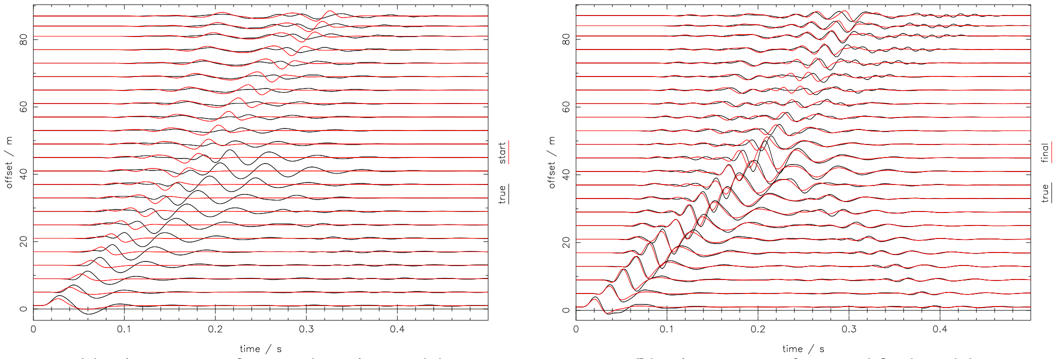
(b) v_s starting model

(c) v_s final model

Figure 6: a) true, b) starting and c) final model for layer with a step assuming a line source seismogram. Sources and receivers are denoted by * and +, respectively. The dashed line indicates the contour of the layer.

Next we want to test the inversion of a 2D structure with point source observed data. We use the same true and starting model as for the line-source-wavefield-inversion (see Figures 6(a) and 6(b)). Before we start the full waveform inversion, we have to correct the amplitudes by the amplitude-only transformation, but no phase-transformation is used. In Figure 8 the final subsurface model is plotted as well as in Figure 9 the comparison between the true, starting and final seismograms. The inversion result for a 2D structure assuming amplitude corrected point source wavefields is not as good as with line source wavefields as observation. The contour of the layer with the step is not really resolved as well as the velocity of the halfspace.

To sum up the effects assuming a point source instead of a line source during a 2D FWI, we would like to mention two points. First the phase shift between seismograms of a point and a line source is reduced during a FWI but yields to artifacts in the subsurface model (e.g. 2D structures within the layer). Second we have to keep in mind that it is not possible to do a multistage approach with frequency filtering for point source seismograms yet. Therefore it's more likely that the FWI ends up in a local minimum as with line source seismograms. With this experience we recommend that amplitude-only transformation is required but also a 3D/2D phase-transformation is reasonable concerning better resolution and more reliable final models. In the following we introduce some phase-transformation techniques and compare them for surface waves.



(a) seismograms of true and starting model **(b)** seismograms of true and final model

Figure 7: Comparison of the line source wavefields (vertical component) for shot at x=6 m between the true, starting and final model; not all traces are shown. The seismograms are not trace normalized but both are scaled by an offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.25}$.

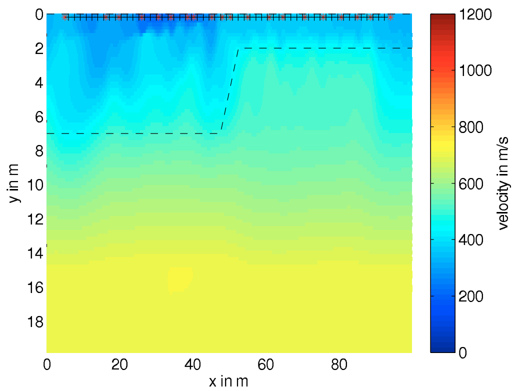
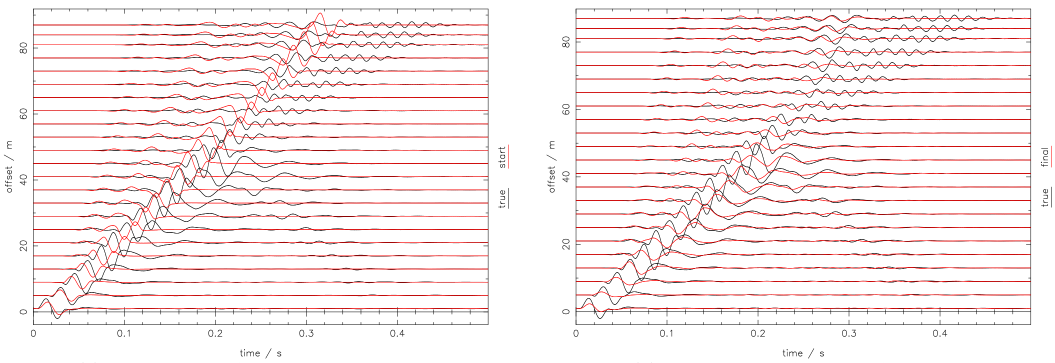


Figure 8: Final model for layer with a step assuming amplitude corrected point source seismograms as observation. Sources and receivers are denoted by * and +, respectively. The dashed line indicates the contour of the layer.



(a) seismograms of true and starting model **(b)** seismograms of true and final model

Figure 9: Comparison of the point source wavefields (vertical component) for shot x=6 m between the true, starting and final model; not all traces are shown. The amplitudes of the point source seismogram are corrected by the amplitude-only transformation (no phase-transformation is used). The seismograms are not trace normalized but both are scaled by an offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.25}$.

3D/2D TRANSFORMATION TECHNIQUES

The results in the previous section have shown that a transformation of point source wavefields to line source wavefields in both amplitude and phase is required before the application of a 2D FWI to point source seismograms. Therefore we are now describing possible transformation techniques and compare them for surface waves. In literature there are already some 3D/2D transformations proposed and successfully applied to body waves. We applied these transformations to shallow seismic surface waves and also considered the simulation of a line source during the measurement.

Transformation 1: Convolution with $t^{-1/2}$

A very simple transformation is suggested by Pica et al. (1990) and Crase et al. (1990). To correct the amplitudes the point source seismograms are first multiplied by $t^{1/2}$ where t is the recording time. Afterwards the time series are convolved with $t^{-1/2}$ to correct for the phase differences between point sources and line sources. When we apply this transformation to surface waves the amplitude correction fails because it is adjusted to body waves and does not consider the different geometrical spreading of surface waves. Therefore we just correct the phases by a convolution between the point source seismograms and the function $t^{-1/2}$. For the amplitude correction we use an offset dependent factor according to the amplitude-only transformation. The two parameters A and x are again determined by a least squares inversion.

Transformation 2: Transformation using the Fourier-Bessel-expansion

This transformation is exact for 1D media and it is suggested by Wapenaar et al. (1992) and Amundsen and Reitan (1994). Given a vertical point force as a source in the origin of the coordinate system and a receiver profile along the horizontal y-axis. For 1D media we can use a Fourier-Bessel-expansion to express the vertical component of the wavefield by

$$\tilde{u}_P(r,\omega) = \int_0^\infty G(\omega,p) J_0(\omega pr)\omega^2 p dp \tag{3}$$

with the Fourier transform $\tilde{u}_P$ of the excited wavefield, the expansion coefficients G, the slowness p, the source-receiver distance r, the Bessel function J_0 of order zero and the angular frequency ω. The seismograms of a line source in a distance y to the line source can be written as a superposition of seismograms excited by an infinite number of point sources along the x-axis. Therefore we obtain

$$\tilde{u}_L(y,\omega) = \int_{-\infty}^\infty \tilde{u}_P(\sqrt{x^2+y^2},\omega)\frac{dx}{[x]} \tag{4}$$

for the Fourier coefficients of a seismogram $\tilde{u}_L(y,\omega)$ excited by a line source along the x-axis. Inserting equation (3) into equation (4) we obtain after some calculation steps

$$\tilde{u}_L(y,\omega) = 2\int_0^\infty G(\omega,p)\cos(\omega py)\omega dp \tag{5}$$

for the line source seismograms. Summarizing this transformation we first have to calculate the expansion coefficients G from the point source seismograms by the back transformation of equation (3) via

$$G(\omega,p) = \int_0^\infty \tilde{u}_p(r,\omega) J_0(\omega pr) r dr. \tag{6}$$

Afterwards we have to do an expansion with plane waves according to equation (5). The transformation for the radial component can be derived in an analog way.

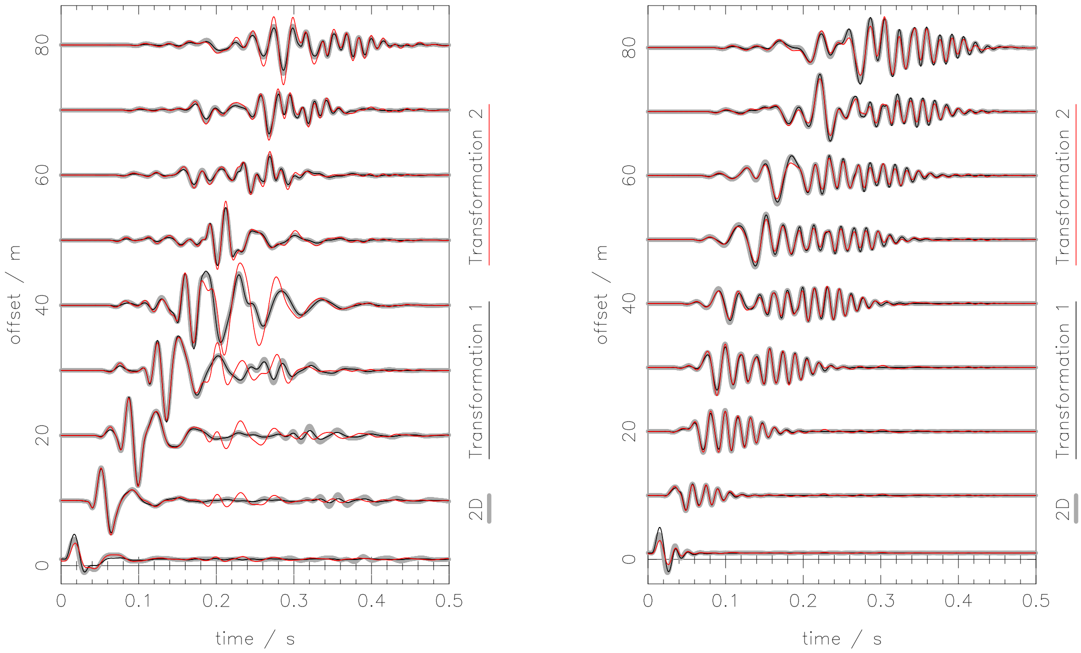
Simulation of a line source

Beside the two transformations known from literature we also consider the simulation of a line source during shallow seismic measurements. In a 2D forward modeling we implicitly use line sources and according to equation (4) a line source can be simulated by the superposition of an infinite number of point sources along a line. Thus we can record seismograms along the y-axis for point sources along the x-axis. The line source seismograms are obtained by the superposition of the seismograms at equal offsets y to the line source. Under the assumption that we use an infinite number of sources this method would provide exact line source seismograms also for 2D subsurface structures. But in practice we have to approximate the integral in equation (4) by a discrete sum of a finite number of point source seismograms. Our tests with synthetic data and field data (on a 1D structure) show that this strategy in principle works but it requires too many point sources to approximate the line source appropriately. The length of the line source as well as the distance between the single point sources influences the offset region and the bandwith for which we obtain satisfying results. For a shallow seismic test example we would need about 80 point sources with a distance of 1 m to obtain line source seismograms up to an offset of 70 m. Thus in practice, the effort during the measurements is too high. Therefore we will not further consider this strategy.

Comparison of the transformation results

To compare the Transformation 1 and 2 we used the 2D subsurface model shown in Figure 6(a) with the two shots at $x_1 = 6\,\text{m}$ (left edge of the profile) and $x_2 = 94\,\text{m}$ (right edge of the profile). Figure 10 shows the results for these two shots. The line source seismograms are plotted as a reference with thick grey lines. The black seismograms are obtained by using Transformation 1 (convolution with $t^{-1/2}$ and the amplitude-only correction) and the red seismograms are gained by using Transformation 2 (using the Fourier-Bessel-expansion). The geometrical spreading (amplitude variation from trace to trace) is corrected quite well with both transformations. However, considering the amplitude differences within the traces we observe that Transformation 1 works better than Transformation 2 (e. g. offsets larger than 40 m in Figure 10(b)). For very small offsets (see offset of 1 m in Figure 10) both transformations do not work properly because we observe differences in amplitude and in phase. This offset region corresponds to the near field and Transformation 1 does not consider near field effects. The inaccuracies of Transformation 2 in this offset region are caused by the superposition of a cutoff phase with the actual signal. This cutoff phase is caused by the numerical implementation of the transformation. The integral in equation (5) is approximated by a sum up to a finite slowness and therefore a cutoff phase is generated. The influence of the cutoff phase can be reduced by using a larger slowness for the upper limit of the approximation of the integration in equation (5). However, for very small offsets (about 1 m) it can't be avoided with acceptable numerical effort. For offsets between 5 m and 40 m both transformations work quite well for the direct waves that correspond to the 1D structure in the left part and the right part of the model, respectively. However, when we give a closer look to the seismograms of the left shot (Figure 10(a)) we see reflected waves by the step in the boundary between the layer and the halfspace. The phases of these waves are well reconstructed with Transformation 1. However, the amplitudes of the transformed waves are too small. Using Transformation 2 neither amplitudes nor phases of these waves can be transformed correctly because this transformation is only valid for 1D media and therefore cannot explain backpropagating waves. Moreover there is an artificial wave generated by Transformation 2 (see Figure 10(a) between 0.18 s and 0.25 s) that propagates with a very high velocity and superimposes the actual signal. The transition between the two 1D structures in the model is in the offset interval between 40 m and 50 m. In this region and also for larger offsets Transformation 2 works less accurate than Transformation 1. According to these observations we would prefer, at least for 2D structures, the very simple transformation with convolution of $t^{-1/2}$ and the amplitude-only correction. For 1D structures Transformation 2 also performs quite well. Related to a FWI we would use the line source seismogams calculated with the initial model to determine the factors for the amplitude correction within Transformation 1. Because of changes in the model during the inversion and therefore changes in the amplitudes of the line source seismograms these factors are maybe outdated after several inversion steps and thus have to be updated during the inversion. But as the amplitude-only correction works quite robust this should be no problem.

In the example shown here the difference between the line source wavefield and the point source wavefield corrected with Transformation 1 is rather small. The largest differences occur in the amplitudes of the backpropagating waves. Therefore we expect to obtain similar inversion results from a FWI with transformed point source seismograms to the results obtained by the inversion of line source seismograms (Figure 6(c)). Nevertheless one of our further steps is the evalution of this.



(a) Shot at $x_1 = 6$ m (left edge of the profile). **(b)** Shot at $x_1 = 94$ m (right edge of the profile).

Figure 10: Comparison of 3D/2D transformations. The line source wavefield is plotted as a reference with the thick grey line. The seismograms obtained with Transformation 1 are plotted in black and the seismograms obtained with Transformation 2 are plotted in red. The seismograms are not trace normalized. All of them are multiplied by the offset dependent factor $\left(\frac{r}{1\,\mathrm{m}}\right)^{0.4}$.

CONCLUSION AND OUTLOOK

We evaluate the reconstruction of the shear wave velocity model for a 1D structure and a 2D structure by applying a 2D full waveform inversion to shallow seismic surface waves. For line source wavefields we obtain satisfying inversion results with a sharp interface between the layer and the halfspace. However, the seismic velocities of the halfspace are not fully correct. If we apply an amplitude correction to point source wavefields and use them as observed data in the inversion our inverted v_s-models contain more artificial 2D structures than the inversion results obtained with the line source wavefields. However, in this case we are not able to use frequency filtering during the inversion. Furthermore, the layer interfaces are not as sharp as in the inversion of line source wavefields. Therefore we conclude that we have to apply a phase correction to the point source wavefields before applying a 2D FWI. We tested two 3D/2D transformations known from literature. Transformation 1 uses a convolution of $t^{-1/2}$ with the point source seismograms to correct the phase differences between point source and line source. The amplitudes are afterwards corrected by an offset dependent factor which is determined by minimizing the misfit of the rms-amplitudes of point source and line source wavefields by a least squares inversion. Transformation 2 uses a Fourier-Bessel-expansion of the point source wavefields to obtain the expansion coefficients. Afterwards these coefficients are used in an expansion with plane waves. For a 1D medium both transformations work quite accurate. The phases of backpropagating waves can be transformed with Transformation 1 whereas Transformation 2 is not able to reconstruct these waves. Furthermore, Transformation 2 produces artificial waves with very high phase velocities which superimpose the actual signal. Therefore Transformation 1 seems to be more accurate than Transformation 2.

Our long-term objective is the inversion of recorded data. For the inversion of field data the optimal source time function needs to be recovered. We are implementing at the moment the inversion of an optimal wavelet in the inversion code using the method described by Forbriger (2003). Within this method the source time function is determined by a linear least squares inversion. To save computing time we look for an optimal wavelet which must be convolved with the observed data to minimize the least squares misfit between the observed data and the synthetic forward modelled data. The convolution with $t^{-1/2}$ to do the 3D/2D phase-transformation could be linked to the convolution with the optimal source time function. Therefore we maybe don't have to apply an explicit 3D/2D phase-transformation. The results presented in this report are just one of the steps towards an inversion of field data. Further steps are the implementation of viscoelasticity, finding an optimal acquisition geometry and the evaluation of the source time function inversion.

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