A SPATIAL APPROXIMATION FOR THE LI CORRECTION

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ABSTRACT

One reason for the high cost of 3D wave-equation migration is the inversion of large matrices involved. The technique of directional splitting reduces this cost by significantly reducing the dimensions of the matrices to be inverted. Unfortunately, this approach is not accurate and introduces a numerical error in the propagation of the wavefields. The most accepted way in the literature to correct for this error is the Li correction. However, this technique still has a high computational cost due to the need of applying multiple Fourier transforms and their inverses. To see if we can avoid this costly method and find a fix that has roughly the same effect as the Li correction, we investigate its theoretical expression in order to approximate the Fourier transforms. To do so, we use the method of stationary phase. We find a convolutional operator of small support that could be used to make an approximate correction. However, its implementation involves finding the directions of the stationary-phase correction by means of an equation system that has no analytical solution. To further facilitate the correction process, we choose to further reduce the convolution operator to a simple application of a phase-correction factor in space, using the direction of wave propagation as the dominant direction. Numerical experiments with the exact propagation angle show that the so-achieved correction has acceptable quality with considerable reduction in computational cost. However, application of this operator in inhomogeneous media requires the extraction of the propagation angle from the wavefield. In our numerical tests, the correction with angles obtained by phase extraction did not reach the same quality as obtained with the exact angles.

INTRODUCTION

The application of wave-equation migration in three dimensions adds the problem of computational cost to the problems of stability and accuracy. To accelerate FD or FFD migration, a technique known as splitting is frequently used, i.e., the separation of single-step 3D migration into two steps inside 2D planes along the horizontal coordinate axes, usually in the inline and crossline directions (Brown, 1983). When operator splitting is applied to the implicit FD operator, so that the equations are solved alternately in the inline and crossline directions, the scheme is called alternating directions implicit (ADI). This bears the disadvantage of being incorrect for reflectors with high slope, resulting in strong positioning errors of reflectors with dip directions far from the directions of the migration planes. Thus, it generates strong numerical anisotropy. Over the years, several approaches were proposed to remedy this problem. Ristow (1980) suggested (see also Ristow and Rühl, 1997), in addition to migrations 2D axes directions, 2D migration in the diagonal directions. Kitchenside (1988) used phase-shift migration plus extrapolation of the wavefield residual by finite differences to reduce the error caused by splitting. Graves and Clayton (1990) proposed implementing a phase-correction operator using finite differences incorporating a damping function to ensure stability of the 3D FD migration scheme.

Instead of using phase-shift migration plus FD residual-wavefield extrapolation as in Kitchenside (1988), Li (1991) proposed the use of conventional FD migration plus residual-wavefield extrapolation by phase shift to improve the quality of the migrated image. Without change in conventional 3D FD migration, the Li correction consists of adding an error compensation by means of a phase-shift filter at certain steps of downward extrapolation. This method not only compensates for the splitting error of extrapolation, but also corrects the positioning error of steeply dipping reflectors.

However, the Li correction is a technique that demands a high computational cost due to the need of applying multiple Fourier transforms and their inverses. However, a large spacing between two subsequent Li corrections is generally not an option because the correction becomes increasingly worse if too much error is accumulated. Therefore, we are interested in a cheaper, possibly approximate, version of the Li correction that can be applied at every depth step without the need for a Fourier transform and its inverse. Such a procedure should help to reduce the high cost of the Li correction while retaining approximately the same effect. To find such an approximation to the Li correction, we investigate its asymptotic behavior by means of the method of stationary phase in order to approximate the Fourier transforms involved.

CORRECTION FOR SPLITTING IN TWO DIRECTIONS (LI CORRECTION)

As discussed previously, splitting in two directions causes numerical anisotropy, i.e., the occurrence of positioning errors in the images of large complex structures. To compensate for these errors and still preserve the efficiency of the method of finite-differences migration, Li (1991) proposed the application of a phase-correction operator, implemented either using the phase-shift method or phase-shift plus interpolation (PSPI). This operator is obtained by evaluating the difference between the ideal and split migration operators.

The idea of this method is to carry out the split migration, i.e., conventional 2D FD migrations in the two coordinate directions, together with a further extrapolation of the wavefield residual. This latter correction is done by the phase-shift method when the lateral velocity variation is small, and by phase-shift plus interpolation when the lateral velocity variation is relevant.

To obtain the Li correction for the techniques discussed above, we expand the one-way wave equation in a complex Padé series and apply the splitting technique in two directions. Rewriting the directional parts of the migration operator in fractional terms, we arrive at the FD migration operator

$$Op_{\text{mig}} = 1 + \sum_{n=1}^{N} \frac{A_n \frac{\partial^2}{\partial x^2}}{1 + B_n \frac{\partial^2}{\partial x^2}} + \sum_{n=1}^{N} \frac{A_n \frac{\partial^2}{\partial y^2}}{1 + B_n \frac{\partial^2}{\partial y^2}} \ . \tag{1}$$

In this work, we consider the complex Padé version of FD migration (Amazonas et al., 2007), where A_n and B_n are the complex Padé coefficients. The error caused by a migration using this operator can be described by the difference

$$\begin{aligned} Op_{\text{dif}} \quad &= \quad \sqrt{1 - \frac{c^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)} \\ &\quad - \left[1 + \sum_{n=1}^{N} \frac{A_n \frac{\partial^2}{\partial x^2}}{1 + B_n \frac{\partial^2}{\partial x^2}} + \sum_{n=1}^{N} \frac{A_n \frac{\partial^2}{\partial y^2}}{1 + B_n \frac{\partial^2}{\partial y^2}}\right] \ . \end{aligned} \tag{2}$$

The above difference operator (2) defines a differential equation that needs to be solved to correct the extrapolated wavefield for the splitting error. It reads

$$\frac{\partial P}{\partial z} = \left[\frac{i\omega}{c(\mathbf{x})} Op_{\text{dif}}\right] P \ . \tag{3}$$

In order to allow for the solution of this differential equation (3) and find the correction operator of Li (1991), the square root in the difference (2) needs to be approximated by means of a Padé expansion. The solution to the differential equation with the resulting approximate difference operator can be represented by means of the finite-difference method as

$$P(z + \Delta z) = e^{i\left(k_z^r - \frac{\omega}{c_r}\right)\Delta z} \prod_{n=1}^{N} \frac{p_n + iq_n}{p_n - iq_n} \prod_{n=1}^{N} \frac{r_n + is_n}{r_n - is_n} P(z) \ , \tag{4}$$

where the coefficients are given by

$$
\begin{aligned}
k_z^r &= \sqrt{\left(\frac{\omega}{c_r}\right) - k_x^2 - k_y^2}, \\
p_n &= \frac{1 - \left[\mu\Delta x^2 + B_n\left(\frac{c_r}{\omega}\right)^2\right]\bar{k}_x^2}{\Delta z}, \\
q_n &= \frac{c_r A_n \bar{k}_x^2}{2\omega}, \\
r_n &= \frac{1 - \left[\mu\Delta y^2 + B_n\left(\frac{c_r}{\omega}\right)^2\right]\bar{k}_y^2}{\Delta z}, \\
s_n &= \frac{c_r A_n \bar{k}_y^2}{2\omega}.
\end{aligned}
$$

In these equations, c_r is a reference velocity and μ is the term of the Douglas (1962) that increases the order of the approximation from second to fourth order. Finally, the overlined symbols $\bar{k}_x$ and $\bar{k}_y$ denote the numerical approximations to the wavenumbers. According to Claerbout (1985), they are given by

$$
\begin{aligned}
\bar{k}_x^2 &= \frac{2 - 2\cos(k_x\Delta x)}{\Delta x^2}, \\
\bar{k}_y^2 &= \frac{2 - 2\cos(k_y\Delta y)}{\Delta y^2}.
\end{aligned}
$$

As is evident from the dependence on the wavenumbers k_x and k_y, the Li correction needs to be applied in the wavenumber domain. This implies that multiple spacial Fourier transforms are required, because the original FD migration operator (1) is in the space domain.

SPATIAL APPROXIMATION OF THE LI CORRECTION

To reduce the cost involved in this procedure, we search for a spatial-domain approximation to the Li correction. For this purpose, we study the stationary-phase evaluation of the involved Fourier transforms.

In the wavenumber domain, the Li correction can be represented by a simple multiplication of the wavefield by a phase-correction factor, given by

$$P_{corr}(k_x, k_y, z, \omega) = P(k_x, k_y, z, \omega)e^{i\frac{\omega}{c_r}E\Delta z}, \tag{5}$$

where the phase-correction term, E, can be written as (Li, 1991, equation 11)

$$
\begin{aligned}
E &= \sqrt{1-\cos^2\phi\sin^2\theta-\sin^2\phi\sin^2\theta} \\
&\quad -\sqrt{1-\cos^2\phi\sin^2\theta} - \sqrt{1-\sin^2\phi\sin^2\theta} + 1 \\
&= \cos\theta - \sqrt{1-\cos^2\phi\sin^2\theta} - \sqrt{1-\sin^2\phi\sin^2\theta} + 1,
\end{aligned} \tag{6}
$$

and where Δz represents the depth interval over which the Li correction is applied. The angles θ and ϕ are the propagation angles for each wavenumber vector component.

After inverse Fourier transform in k_x and k_y, we obtain

$$P_{corr}(x, y, z, \omega) = \int dk_x dk_y P(k_x, k_y, z, \omega)e^{i\frac{\omega}{c_r}E\Delta z}e^{ik_x x + ik_y y}. \tag{7}$$

To find a computationally more economic correction procedure, we apply certain approximations to this expression for the case of a homogeneous medium, i.e., where c_r is the constant reference velocity.

Approximation by convolution

Expressing the wavefield $P(k_x, k_y, z, \omega)$ in the wavenumber domain by means of the Fourier transform, we obtain

$$\begin{aligned} P_{corr}(x,y,z,\omega) &= \int dk_x dk_y \int dx'dy'\; P(x',y',z,\omega) \\ &\quad \times e^{i\frac{\omega}{c_r}E\Delta z} e^{ik_x(x-x')+ik_y(y-y')} \\ &= \int dx'dy'\; P(x',y',z,\omega) \\ &\quad \times \underbrace{\int dk_x dk_y e^{i\frac{\omega}{c_r}E\Delta z} e^{ik_x(x-x')+ik_y(y-y')}}_{C(x-x',y-y')}\,, \end{aligned} \tag{8}$$

where we have changed the order of integration to arrive at the final expression. Note that the operation described by this equation is a convolution with the operator C resulting from the internal integration.

Stationary phase To simplify the convolutional operator C, we approximate its integral expression by means of the stationary-phase method. This leads to

$$\begin{aligned} C(x-x',y-y') &= \int dk_x dk_y\, e^{i\frac{\omega}{c_r}E\Delta z} e^{ik_x(x-x')+ik_y(y-y')} \\ &\approx \sqrt{\frac{c_r}{\omega \det \mathbf{E}\Delta z}} e^{i\frac{\omega}{c_r}E^*\Delta z} e^{ik_x^*(x-x')+ik_y^*(y-y')}, \end{aligned} \tag{9}$$

where $\mathbf{E}$ denotes the Hessian matrix of second derivatives of E with respect to k_x and k_y, and E^* denotes the value of E at the stationary point. This point is defined by the stationary values k_x^* e k_y^*, which in turn are defined by

$$\begin{aligned} &\nabla_k \left(\frac{\omega}{c_r} E\Delta z + k_x(x-x') + k_y(y-y') \right)\Bigg|_{k_x^*,k_y^*} \\ &\qquad = \left(\frac{\omega}{c_r} \Delta z \nabla_k E + \Delta\vec{x} \right)\Bigg|_{k_x^*,k_y^*} = \vec{0}\,, \end{aligned} \tag{10}$$

where $\nabla_k = (\partial_{k_x}, \partial_{k_y})$ denotes the gradient in the horizontal wavenumber components and $\Delta\vec{x} = (x - x', y - y')$. The assumption of a homogeneous medium leads to the second expression in equation (10).

Stationary directions. To calculate the derivatives $\partial E/\partial k_i$ ($i = x, y$), we make use of the dependency of the components k_i on the propagation angles ϕ and θ. By means of the chain rule, we can write

$$\frac{\partial E}{\partial k_i} = \frac{\partial E}{\partial \theta}\frac{\partial \theta}{\partial k_i} + \frac{\partial E}{\partial \phi}\frac{\partial \phi}{\partial k_i}\,. \tag{11}$$

Differentiating E of equation (6) with respect to θ and ϕ, we find

$$\frac{\partial E}{\partial \theta} = -\sin\theta + \frac{\cos^2\phi\sin\theta\cos\theta}{\sqrt{1-\cos^2\phi\sin^2\theta}} + \frac{\sin^2\phi\sin\theta\cos\theta}{\sqrt{1-\sin^2\phi\sin^2\theta}}, \tag{12}$$

$$\frac{\partial E}{\partial \phi} = -\frac{\cos\phi\sin\phi\sin^2\theta}{\sqrt{1-\cos^2\phi\sin^2\theta}} + \frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{1-\sin^2\phi\sin^2\theta}}\,. \tag{13}$$

The derivatives of the propagation angles with respect to the components of the wavenumber vector can be found from the equations relating these quantities, i.e.,

$$k_x = \frac{\omega}{c_r}\sin\theta\cos\phi\,, \tag{14}$$

$$k_y = \frac{\omega}{c_r}\sin\theta\sin\phi\,. \tag{15}$$

Differentiating these equations with respect to k_x and k_y, respectively, we find

$$1 = \frac{\omega}{c_r}\left(\cos\theta\cos\phi\frac{\partial\theta}{\partial k_x} - \sin\theta\sin\phi\frac{\partial\phi}{\partial k_x}\right) , \tag{16}$$

$$0 = \frac{\omega}{c_r}\left(\cos\theta\cos\phi\frac{\partial\theta}{\partial k_y} - \sin\theta\sin\phi\frac{\partial\phi}{\partial k_y}\right) , \tag{17}$$

$$0 = \frac{\omega}{c_r}\left(\cos\theta\sin\phi\frac{\partial\theta}{\partial k_x} + \sin\theta\cos\phi\frac{\partial\phi}{\partial k_x}\right) , \tag{18}$$

$$1 = \frac{\omega}{c_r}\left(\cos\theta\sin\phi\frac{\partial\theta}{\partial k_y} + \sin\theta\cos\phi\frac{\partial\phi}{\partial k_y}\right) . \tag{19}$$

Multiplication of equation (16) with $\cos\phi$ and of equation (18) with $\sin\phi$ and summation of the resulting equations leads to

$$\cos\phi = \frac{\omega}{c_r}\cos\theta\frac{\partial\theta}{\partial k_x} . \tag{20}$$

Correspondingly, multiplication of equation (17) with $\cos\phi$ and of equation (19) with $\sin\phi$ and summation of the resulting equations leads to

$$\sin\phi = \frac{\omega}{c_r}\cos\theta\frac{\partial\theta}{\partial k_y} . \tag{21}$$

By exchanging in these operations $\sin\phi$ by $\cos\phi$ and $\cos\phi$ by $-\sin\phi$, we find analogously

$$-\sin\phi = \frac{\omega}{c_r}\sin\theta\frac{\partial\phi}{\partial k_x} , \tag{22}$$

$$\cos\phi = \frac{\omega}{c_r}\sin\theta\frac{\partial\phi}{\partial k_y} . \tag{23}$$

Combining these equations, the searched-for derivatives of the propagation angles with respect to the wavenumber components can be represented as

$$\frac{\partial\theta}{\partial k_x} = \frac{c_r}{\omega}\frac{\cos\phi}{\cos\theta} , \tag{24}$$

$$\frac{\partial\theta}{\partial k_y} = \frac{c_r}{\omega}\frac{\sin\phi}{\cos\theta} , \tag{25}$$

$$\frac{\partial\phi}{\partial k_x} = -\frac{c_r}{\omega}\frac{\sin\phi}{\sin\theta} , \tag{26}$$

$$\frac{\partial\phi}{\partial k_y} = \frac{c_r}{\omega}\frac{\cos\phi}{\sin\theta} . \tag{27}$$

Substitution of these expressions, together with equations (12) and (13), in equation (11), yields

$$\frac{\partial E}{\partial k_x} = \frac{c_r}{\omega}\cos\phi\tan\theta\left(\frac{\cos\theta}{\sqrt{1-\cos^2\phi\sin^2\theta}} - 1\right) , \tag{28}$$

$$\frac{\partial E}{\partial k_y} = \frac{c_r}{\omega}\sin\phi\tan\theta\left(\frac{\cos\theta}{\sqrt{1-\sin^2\phi\sin^2\theta}} - 1\right) . \tag{29}$$

Upon using these equations in the stationary-phase equation (10), the propagation angles that define the stationary direction are given by the equation system

$$x - x' = -\cos\phi\sin\theta\left(\frac{1}{\sqrt{1-\cos^2\phi\sin^2\theta}} - \frac{1}{\cos\theta}\right)\Delta z, \tag{30}$$

$$y - y' = -\sin\phi\sin\theta\left(\frac{1}{\sqrt{1-\sin^2\phi\sin^2\theta}} - \frac{1}{\cos\theta}\right)\Delta z. \tag{31}$$

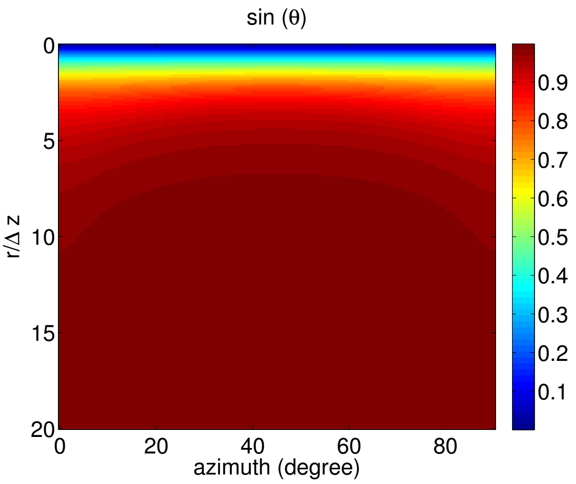

Figure 1: Stationary value of $\sin\theta$ as obtained by solving system (33-34) by means of Newton's method.

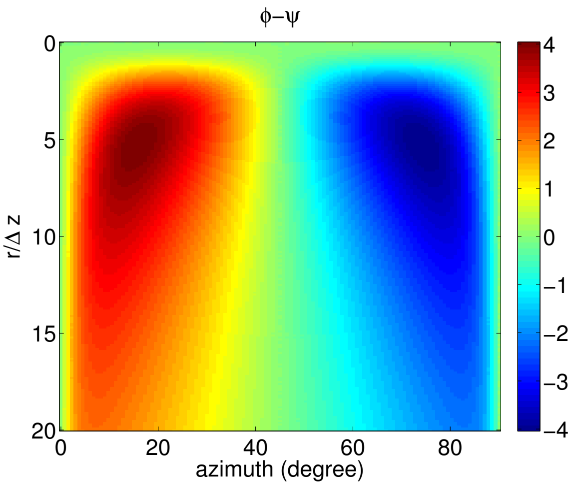

Figure 2: Stationary value of the difference $\phi - \psi$ as obtained by solving system (33-34) by means of Newton's method.

Propagation direction. Representing the displacement vector $\Delta\vec{x}$ in polar coordinates, i.e.,

$$x - x' = r\cos\psi \quad \text{and} \quad y - y' = r\sin\psi\ , \tag{32}$$

we can recast system (30-31) into the form

$$\cos\phi\sin\theta\left(\frac{1}{\sqrt{1-\cos^2\phi\sin^2\theta}} - \frac{1}{\cos\theta}\right)\Delta z + r\cos\psi \quad = \quad 0\ , \tag{33}$$

$$\sin\phi\sin\theta\left(\frac{1}{\sqrt{1-\sin^2\phi\sin^2\theta}} - \frac{1}{\cos\theta}\right)\Delta z + r\sin\psi \quad = \quad 0\ . \tag{34}$$

It is easy to verify that this system is satisfied in the following horizontal dislocation directions ψ

$$\psi = 0^\circ : \qquad \phi = 0^\circ\ , \tag{35}$$

$$\psi = 90^\circ : \qquad \phi = 90^\circ\ , \tag{36}$$

$$\psi = 45^\circ : \qquad \phi = 45^\circ\ . \tag{37}$$

For other directions ψ, system (33-34) needs to be solved numerically. Since this system does not depend on ω or c_r, it can be solved independently of the actual migration problem to be solved, with the stationary values of the directions stored in a table.

When solving system (33-34) for a number of directions ψ and distances r, we noted that the stationary point only exists for rather small distances r. Moreover, the value of $\sin\theta$ is very close to 1 for a great part of the domain under consideration (Figure 1). This means that the stationary direction is close to the horizontal, and the angle ϕ is always relatively close to the dislocation angle ψ, with differences below 4° (Figure 2). This motivated us to abandon the convolutional approach for a even simpler one, discussed below.

Single-point approximation of the correction

Because of these characteristics of the solution to system (33-34), we assume that in the correction equation (7), a single propagation direction is mainly responsible for the outcome. For simplicity, we assume this direction to be well-approximated by the direction parallel to the propagation vector. Apart from the above discussed fact that the deviation between these directions is small for most points in the medium, this direction has also the advantage of being rather easily determined from the wavefield. Thus, the correction factor E is approximated by its value in this direction, i.e., $E \approx E^p = E(\theta = \theta^p, \phi = \phi^p)$. As a consequence, equation (7) can be approximated by

$$\begin{aligned} P_{corr}(x,y,z,\omega) &\approx e^{i\frac{\omega}{c_r}E^p\Delta z}\int dk_x dk_y P(k_x,k_y,z,\omega)e^{ik_x x+ik_y y} \\ &\approx e^{i\frac{\omega}{c_r}E^p\Delta z}P(x,y,z,\omega) \; . \end{aligned} \tag{38}$$

In this way, we arrive at an approximate correction directly in space, the application of which is much simpler than the full Li correction (4) and which does not require the execution of a convolutional operation like equation (8). Please note, however, that this approach is rather crude. An approximate solution of system (33-34) might provide better values of θ and ϕ for each Δz and ψ, even if r is small. Such a procedure will still allow the application of the multiplicative correction (38) with better values for the propagation angles and might thus lead to a better correction than our crude procedure.

Extraction of the propagation angle

The application of the simplified correction (38) demands the knowledge of the present propagation angle of the wavefield to be corrected at a point (x, y). In a homogeneous medium, this angle can be determined from the relative position of the point with respect to the source location. However, in a heterogeneous medium, the angle ψ must be extracted from the wavefield at (x, y). To achieve this extraction, we use the identities

$$\begin{aligned} \nabla^{(h)}P(x,y,z,\omega) &= \nabla^{(h)}\left[P_0(x,y,z)e^{i\omega\tau(x,y,z)}\right] \\ &\approx P_0(x,y,z)e^{i\omega\tau}i\omega\nabla^{(h)}\tau(x,y,z) \; , \end{aligned} \tag{39}$$

where $\nabla^{(h)}$ represents the horizontal components of the gradient vector, i.e., $\nabla^{(h)} = (\partial/\partial x, \partial/\partial y)$. In this way, the horizontal slowness vector is given by

$$\vec{p}^{(h)} = \nabla^{(h)}\tau(x,y,z) \approx \frac{1}{i\omega P(x,y,z,\omega)}\nabla^{(h)}P(x,y,z,\omega) \; . \tag{40}$$

Since this vector also has to satisfy

$$\vec{p}^{(h)} = \frac{1}{c_r}\begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} \; , \tag{41}$$

we can conclude that

$$\begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} \approx \frac{c_r}{\omega}\Im\left\{\frac{\nabla^{(h)}P(x,y,z,\omega)}{P(x,y,z,\omega)}\right\} \; . \tag{42}$$

We stress that this is a high-frequency approximation. For lower frequencies, the propagation angle extracted in this way might not correctly represent the true propagation direction of the wavefield under consideration.

NUMERICAL RESULTS

We tested the approximate correction (38) for wave propagation in a homogeneous model. Figure 3 shows four horizontal slices through the impulse response of FD migration in a homogeneous medium without Li correction. The same slices after conventional Li correction at every 6th depth level are shown in Figure 4. Note the improvement of the circular shape, particularly at the intermediate depth levels.

The corresponding results using the approximate Li correction of equation (38) are shown in the next figures. First, we show the result when the wavefield-propagation angles are calculated from the position of the grid point where the correction is performed (Figure 5). We see that the wavefronts were well corrected, resulting in an almost perfectly circular shape. The correction is of at least the same quality as that of the conventional Li correction (compare to Figure 4). Note, however, that this procedure is only possible in the homogeneous case, where the propagation direction is always in radial direction from the source. We have included these figures to demonstrate the potential of the approximate correction.

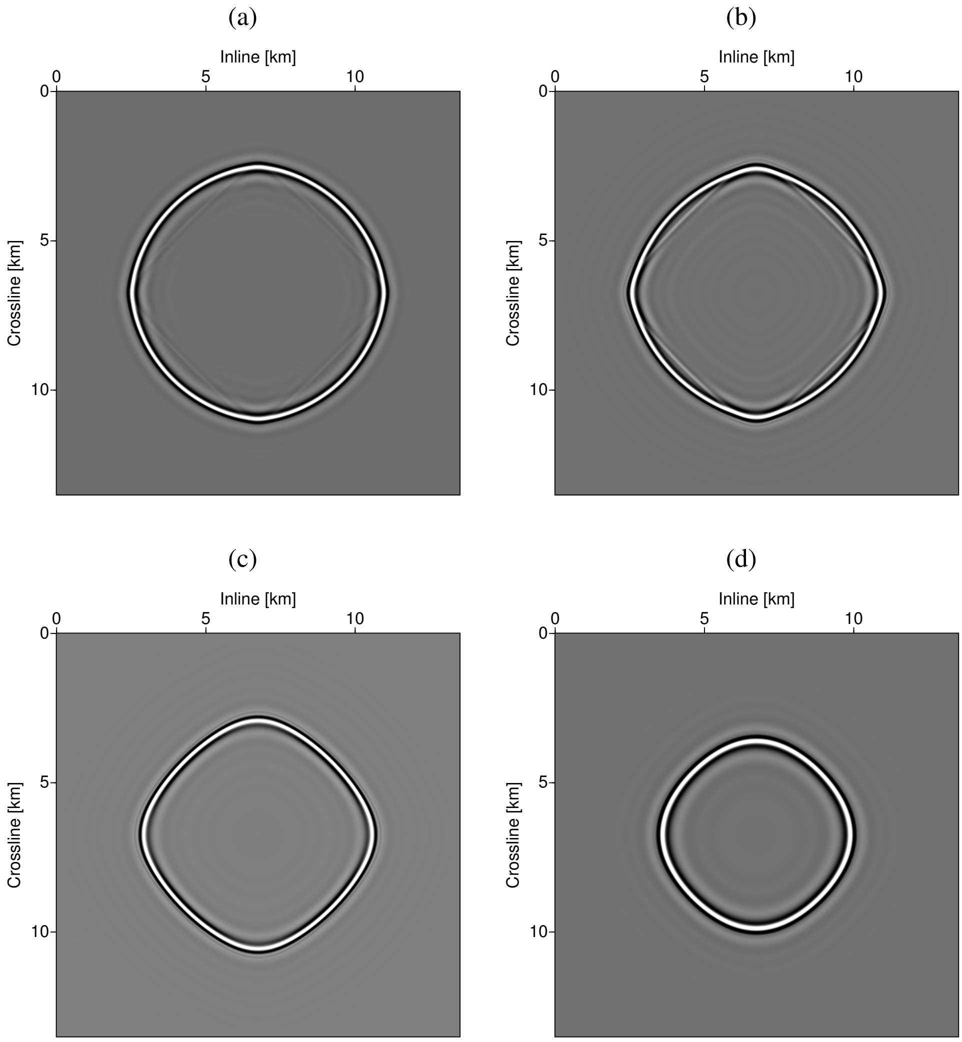

Figure 3: Horizontal slices through the impulse response of FD migration without Li correction in a homogeneous medium at the depths (a) 480 m, (b) 960 m, (c) 1920 m, and (d) 2880 m.

When we extracted the propagation direction directly from the wavefield, frequency by frequency, we obtain the corrected result shown in Figure 6. We note that the quality of the achieved correction is reduced as compared to the result of the grid-position angles (Figure 5). Still, the quality is superior to the result without applying any correction (Figure 3).

We have experimented with various modifications of the algorithm used to extract the propagation angles from the wavefield, notably working with selected frequencies and smoothing over adjacent frequencies. The best result that was obtained by application of such techniques is presented in Figure 7. In this case, the phase extracted from the wavefield was averaged over six adjacent frequencies, while the horizontal gradient field was smoothed over 10 neighboring points. We observe an improvement compared to the results of Figure 6, but even smoothing could help to not reach the same quality as in Figure 5. We note specifically that the smoothing created some artifacts inside the wavefront at greater depth (see Figure 7d).

As another way of improving the extracted propagation angles, we experimented with different implementations of the numerical derivative. The best quality was achieved with a Gaussian derivative or with the centered derivative of the mean between adjacent field values. Moreover, we chose at each point in the image the angle that was extracted from that frequency component which had the highest amplitude at

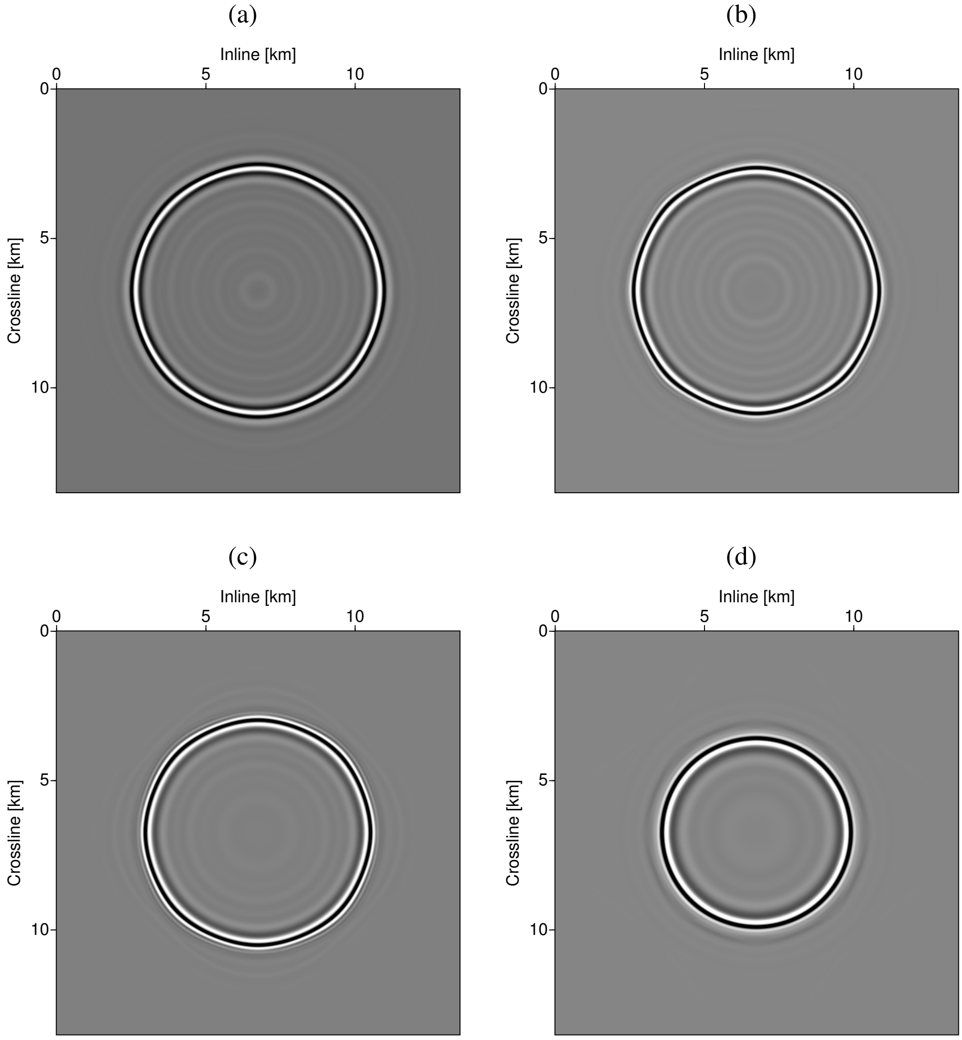

Figure 4: Corresponding slices to Figure 3 after conventional Li correction.

that point. This reduces the sensitivity of the wavefield derivative on numerical noise in the data. The best correction obtained with angles extracted from these tests is depicted in Figure 8. While the circular shape is almost as well recovered as by the correction with the grid angles (see Figure 5), the artifacts are even stronger than in Figure 7.

These tests, particularly the one with the grid angles, indicate that approximation (38) is of acceptable quality. However, further tests to improve the extraction of the propagation angle, particularly in inhomogeneous media, are required to make sure the best possible correction is achieved. We cannot rule out the possibility that the numerical realization of approximation (42) does not have enough quality to determine the propagation angle with the necessary precision for the wavefield under consideration, particularly when applied to noisy data.

Computational cost

The elimination of the back-and-fourth Fast Fourier transform for each application of the Li correction leads to a significant reduction in computation time needed for the wavefield correction. In our experiments, we compared the application of the conventional Li correction at every 8th depth level with the application of the approximate spatial Li correction at every depth level. In our implementation and under these

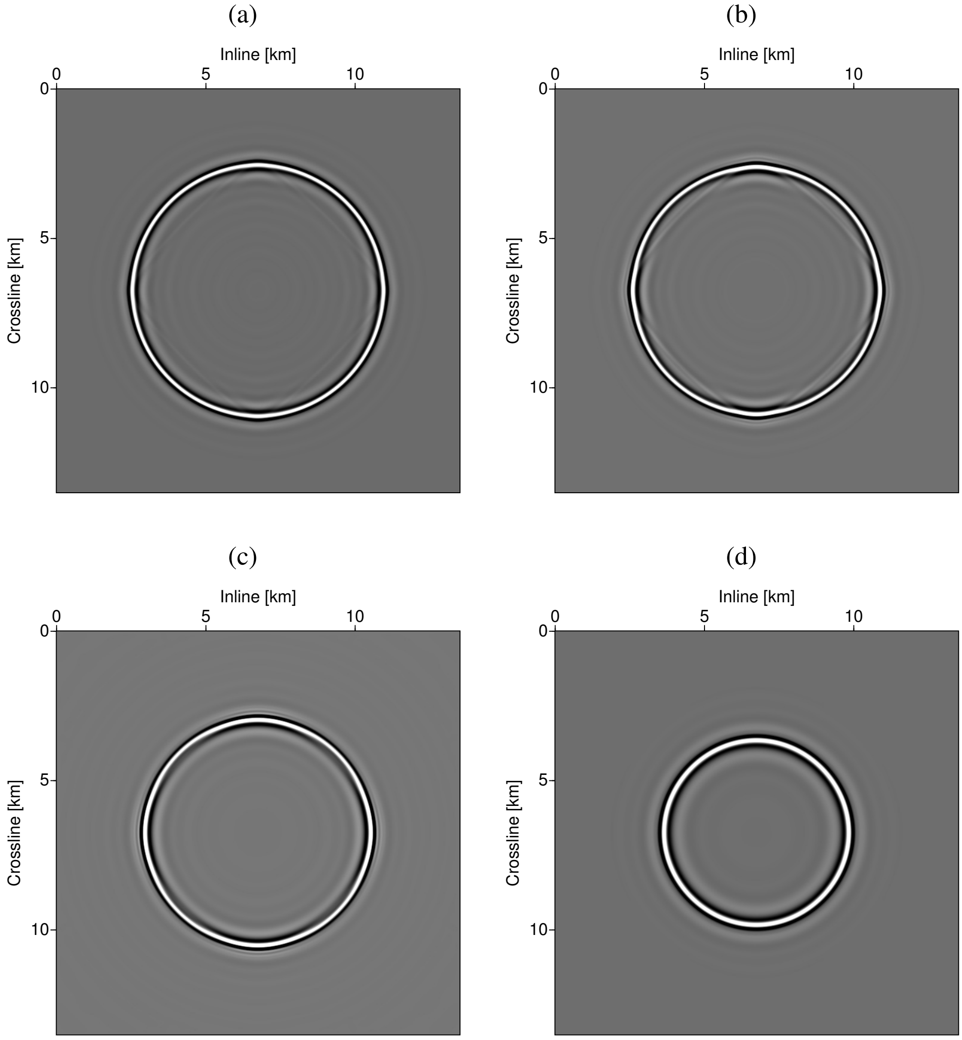

Figure 5: Corresponding slices to Figure 3 after approximate Li correction (38) using the angles calculated from the grid position.

conditions, the approximate spatial Li correction was about a factor of 4 times faster than the conventional correction. The different types of extracting the propagation angle did not result in significant differences in the required computation time.

CONCLUSIONS

It is well-known that the implementation of three-dimensional migration by means of the directional-splitting technique causes numerical anisotropy. The most widely used method to reduce this effect is called Li correction (Li, 1991). However, the Li correction is a technique that still has a relatively high computational cost due to the need of applying multiple Fourier transforms and their inverses. To see if it is possible to reduce this cost, we have tried to find an approximate correction that has roughly the same effect as the Li correction. For this purpose, we investigate the theoretical expression of the latter in order to approximate the involved Fourier transforms. For this analysis, we utilized the method of stationary phase applied to the Li correction in a homogeneous medium.

We found a convolutional operator of small support that could be used to make an approximate correction. However, its implementation involves finding the directions of the stationary-phase correction through

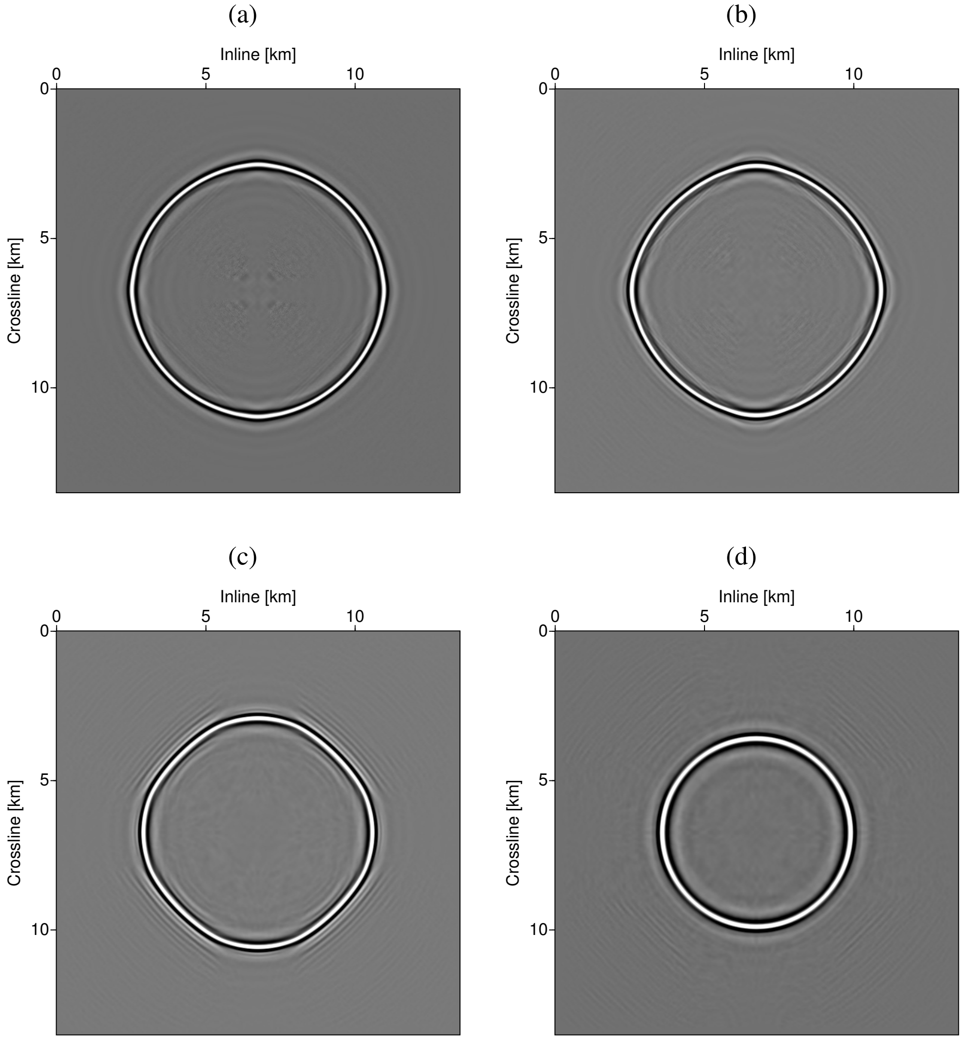

Figure 6: Corresponding slices of Figure 3 with approximate Li correction (38) using angles extracted from the propagating wavefield at every frequency.

a system of equations that has no analytical solution. Since these equations do not depend on the signal frequency and the value of the supposedly constant velocity, the system could be resolved in principle once and for all, saving the stationary directions for each direction in a table.

However, the correction operation would still be a somewhat expensive convolution. To further simplify and cheapen the correction process, we chose to reduce the convolution operator to a single pointwise application of an approximate phase-correction factor in space. From a trial solution of the stationary-phase equations, we know that the stationary angles are close to the dominant propagation direction of the wavefield. Numerical experiments with the exact propagation angle calculated from the grid position show that the approximate correction achieved by this operator has acceptable quality and can achieve a considerable reduction in computational cost. However, the application of this approximate correction factor in inhomogeneous media requires the extraction of the propagation angle directly from the wavefield. In our numerical tests, the correction with angles obtained by such an extraction did not reach the same quality as obtained by the exact angles. This suggests that if a better extraction technique can be found, the approximate correction can become an interesting alternative to a full Li correction. Because of its lower computation cost, it can be applied at each depth level, avoiding the accumulation of errors over a larger

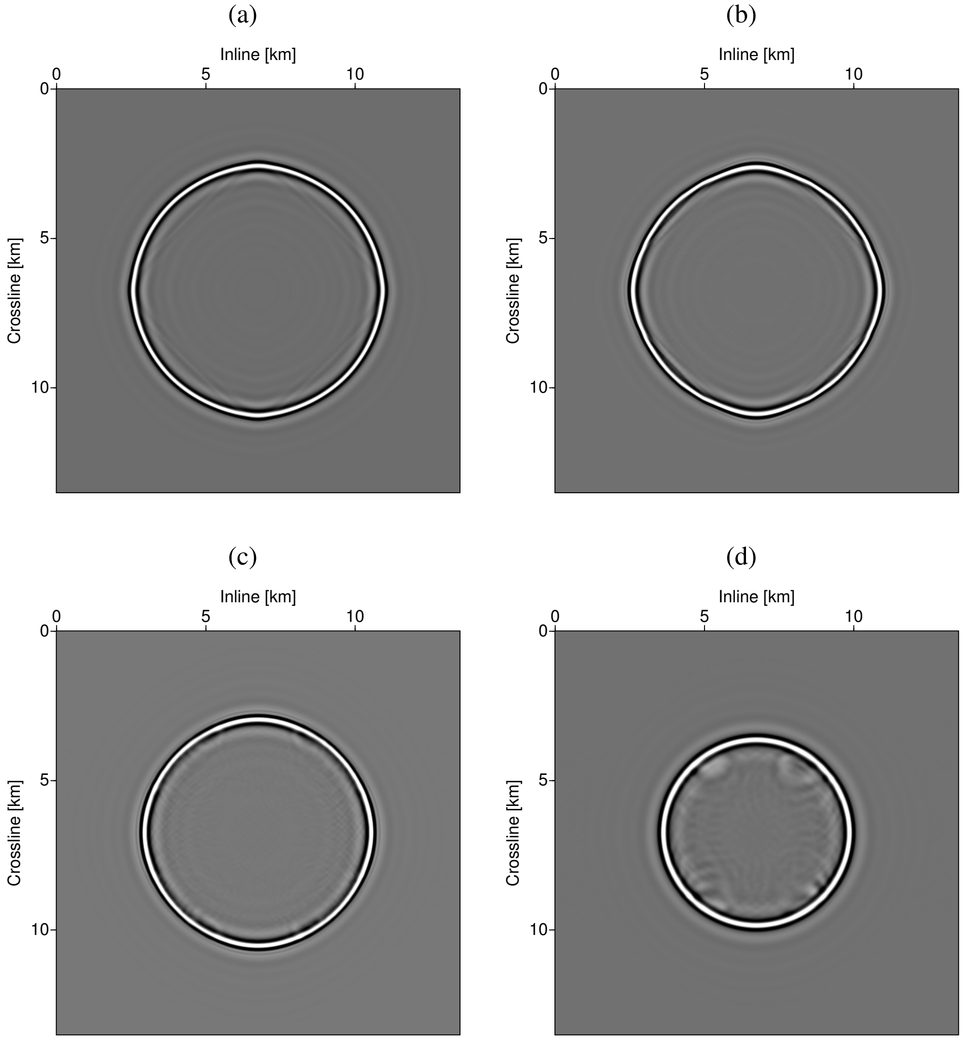

Figure 7: Corresponding slices of Figure 3 with approximate Li correction (38) using angles extracted from the propagating wavefield at 6 frequencies with smoothing.

depth interval.

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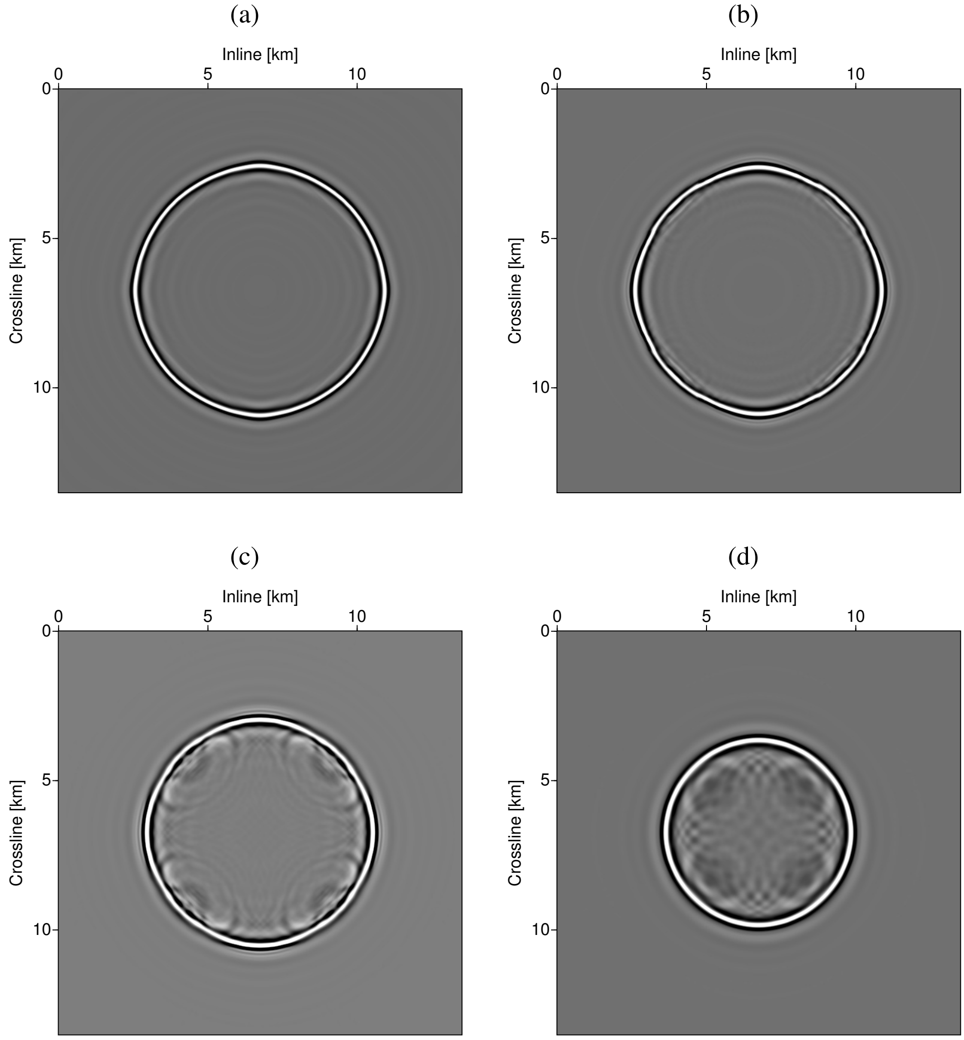

Figure 8: Corresponding slices of Figure 3 with approximate Li correction (38) using angles extracted from the highest-amplitude frequency component of the propagating wavefield at each image point.

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